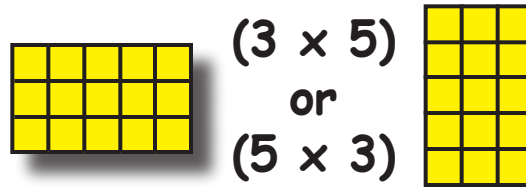
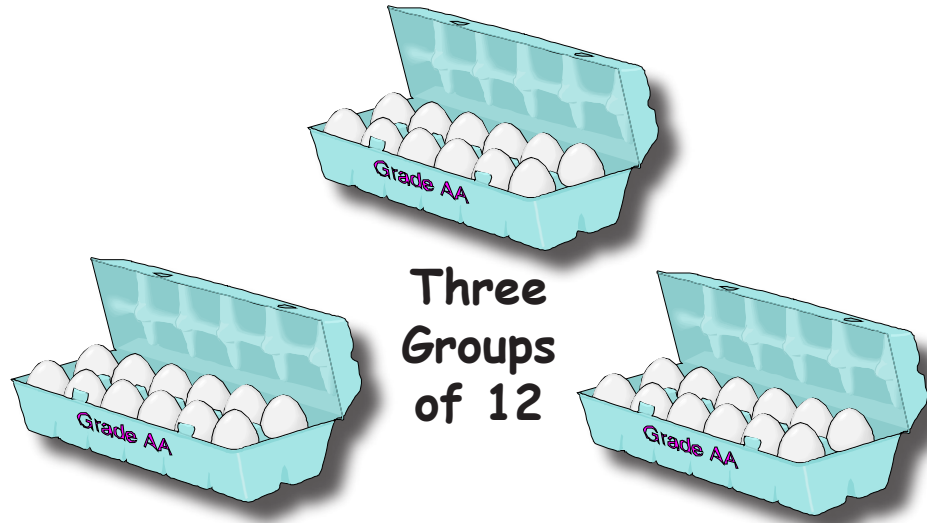


What is Multiplication?



Groups, Rows, and Columns

Outcomes

- To develop an understanding of multiplication.
- To use arrays, grouping and repeated addition to help children understand multiplication.
- To reflect on the changes in mathematics education.

Overview

This module is intended to engage participants in understanding the meaning of multiplication and experiencing different models for multiplication. Participants explore multiplication by looking at grouping, repeated addition, and arrays. They think of examples of things that come in groups, and then make up stories with those groups to illustrate multiplication. They then build arrays as a visual representation of multiplication. The arrays are posted so that patterns and big ideas related to multiplication can be discovered. The array model is used to stimulate questions and discussion about divisibility, even and odd numbers, prime numbers and square numbers. Participants go home with some games and activities.

As participants (parents and children) enter, they are asked to think about things that come in groups and record their ideas on charts which are posted around the room. Participants make up story problems using the groups and then discuss the way that the problems can be solved. The discussion of strategies introduces the concept of multiplication as groups of items and as repeated addition.

Participants take some time to reflect on what they know about multiplication. The ideas are shared and recorded. The question is revisited at the end of the session as a way to process the activities of the session.

After discussing multiplication, participants have an opportunity to build arrays representing the numbers through 25. These are displayed in the front of the room for everyone to see. At this point, the children are asked to leave. Experience has shown that parents have more opportunity for discussion during the remaining parts if children are not present. The participants then look at the arrays and make at least three observations. Ideas are recorded. For a discussion of ideas that might be shared, see the facilitator notes or the mathematical background.

Next, the original question, "What is multiplication?" is revisited. Parents make additions or deletions to their original chart. Afterwards, participants are asked to think about how they can use repeated addition, grouping and arrays to help their children learn multiplication.

The session ends with take home activities for parents to use with their children.

Mathematics Background

The mathematical focus of this module is multiplication and the various ways that it can be represented. Three ways are explored: repeated addition, groups of items, and rectangular arrays. These three ways can be illustrated through the multiplication fact $4 \times 5 = 20$.

Repeated Addition

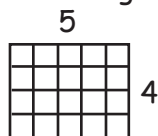
$$5 + 5 + 5 + 5 = 20$$

Groups of Items

4 groups of 5 as in 4 groups of 5 pencils

Array

a rectangle with 4 rows of 5 in each row



Some people may read 4×5 as "4 five times" and would interpret it as $4 + 4 + 4 + 4 + 4$ or as 5 groups of 4. Whether it is interpreted as 4 groups of 5 or 5 groups of 4 the answer is 20. This is an important concept and is called The Commutative Property ($a \times b = b \times a$). The Commutative Property states that the order of the numbers in multiplication does not effect the answer. When multiplication is done in context, interpretation is not an issue. For example, there are 6 children and each one has 3 books. How many books do they have? This gives 6 groups of 3 and not 3 groups of 6.

Repeated Addition

It is important for students to make connections between addition and multiplication. It will help them to understand and learn their multiplication facts. If they know that $4 \times 5 = 20$, then they can use the connection between multiplication and addition to learn that 4×6 is one more 4, or $20 + 4$. Later, students can also use this connection for more complicated problems such as 24×51 . Since 24×51 is one group of 24 more than 24×50 . Some students multiply by 50 first because they realize that is 24×50 plus one more 24. Since 50 is one half of 100, a simple way to multiply by 50 is to multiply by 100 and halve the answer. So, 24×100 (2400) cut in half (1200). Finally, add that additional 24: 1200 plus 24 is 1224.

Skip counting is also associated with multiplication. It is actually a form of repeated addition. To multiply 4×5 , one can count "5, 10, 15, 20". This is counting by fives 4 times.

Groups of Items

Another way to look at multiplication is as groups of items. Using groups helps students understand the connection of multiplication to the real world. Word problems have traditionally been hard for students because they are not able to visualize the meaning of the operations. As students work with groups of items, they are more able to make the connections between multiplication and problems like: Mrs. Jones has 4 bags. Each bag contains 5 cookies. How many cookies does she have?

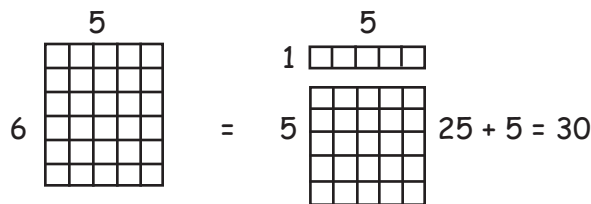
Arrays

Arrays are area models of multiplication and are often used in working with fractions, decimals, percents and algebra. Besides helping students understand the meaning of multiplication, arrays are useful in learning multiplication facts. Students can build an array and break the array into parts to help them get the answer (the product). They can use what they already know to help them find what they do not know. Some examples of this are on the next page.

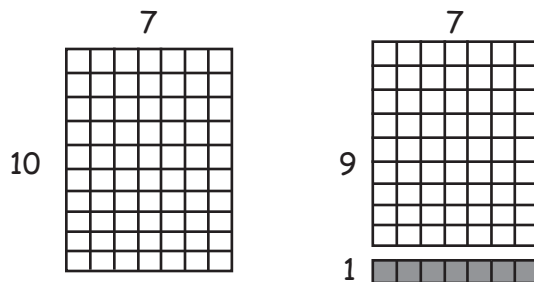
Mathematics Background (continued)

Examples:

- 1) When a 6×5 array is built, students can see that this is a 5×5 array plus an additional 5.



- 2) If a student needs to find the product of 9 and 7 (9 sevens), he/she can build a 10 by 7 array (10 sevens) and then remove one 7 to get to 9 sevens. This is helpful since 10 sevens (70) is usually an easy fact for students. Then, it is a matter of subtracting one group of seven or $70 - 7 = 63$.



Several other ideas may come up when participants are looking at the arrays.

- 1) Prime numbers form only two arrays. (One is not a prime number because it forms only one array).
- 2) Even numbers have an array that can be arranged in two even rows.
- 3) The arrays of odd numbers cannot be arranged in two even rows.
- 4) Square numbers have an odd number of arrays, one of which is a square.
- 5) The lengths and widths of the arrays are the factors of the number.

Factors and Multiples: Background Information

As you are using the term factor to describe other numbers, be sure to write the word factor and give examples of the factors of a number.

- 1) Factors are the numbers which are multiplied together to obtain the product (3 and 4 are factors of 12). A factor of a number divides into that number without leaving a remainder.
- 2) The length of the sides of an array for a number gives the factors of that number. Since 12 has the following arrays: 1×12 , a 2×6 and a 3×4 , the factors of 12 are 1, 2, 3, 4, 6, and 12.
- 3) Multiples of a number are the products of that number and any whole number (multiples of 3 are 0, 3, 6, 9, 12,...)

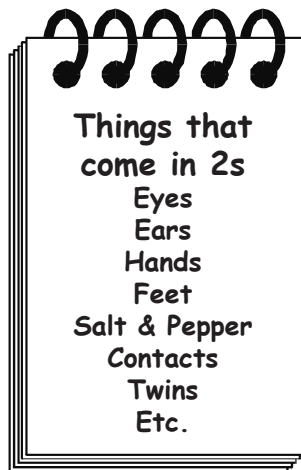
Mathematics Background (continued)

Opening Activity:

The opening activity has participants filling out charts of items that come in groups. Each chart has written on the top "Things that come in...". To clarify what participants are being asked to do, the instructions are written on the overhead projector or on an easel with an example. An example for things that come in 2s would be eyes. Some ideas for each number are listed below.

- 2s: eyes, hands, feet, ears, salt and pepper shakers, wheels on a bike, scooter, or motorcycle, contacts, twins, hands on a clock, shoes, gloves
- 3s: wheels on a tricycle, combo meal, triplets, prongs on an electrical plug, tennis balls, paper towel rolls, periods in wrestling
- 4s: wheels on a car, quarters in a dollar, leaves on a clover, blades on a fan, batteries, seasons, rolls of toilet paper, periods in a football game
- 5s: fingers on one hand, toes on one foot, pennies in a nickel, nickels in a quarter, blades on a fan, sticks of gum, senses
- 6s: soda pop, minutes in a wrestling match
- 7s: days of the week, wonders of the ancient world, number of seas, Snow Whites dwarfs
- 8s: pizza slices, batteries, hot dog buns, octopus's arms, crayons, bars of soap
- 9s: golf balls, baseball innings, squares on tic-tac-toe, nine months of school, planets
- 10s: toes, fingers, pennies in a dime, dimes in a dollar, bowling pins, years in a decade
- 11s: number of football players on a field, number of soccer players on a field
- 12s: eggs, months, soda pop, donuts, bagels

Example Chart:



Room Setup

- Desks or tables arranged in groups of 4-6
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda
- A large wall to hang butcher paper or folders

Materials

Facilitator	Transparencies
<ul style="list-style-type: none"> • Overhead projector • Overhead pens • Transparencies, blank • Overhead square tiles • Chart paper • Chart markers • Masking tape • Timer (optional) • Scissors • Colored construction paper, 11 x 18 (or larger) • Egg carton (optional) • 1 x 12 array cut from grid paper 	<p><i>BLM 1: Welcome</i> <i>BLM 2: Estimation Instructions</i> <i>BLM 4: What is Multiplication?</i> <i>BLM 5: One-Inch Graph Paper</i> <i>BLM 8: NCTM Number and Operations Standard</i></p>
Participant	Handouts
<p>Individuals:</p> <ul style="list-style-type: none"> • Pencil • Paper • 3 index cards, 3x5 • Reflection <p>Groups:</p> <ul style="list-style-type: none"> • Chart paper • Chart markers • Scissors • Masking tape • Tub of square tiles (approx. 40 per person) 	<p>25 copies for each group <i>BLM 5: One-Inch Graph Paper</i> One per participant for class <i>BLM 2: Estimation Instructions</i> <i>BLM 3: Estimation Question Cards</i> (copy on cardstock, cut and prepare) <i>BLM 6: Finding Multiples</i> <i>BLM 7: List of Terms</i> One per participant for home (make packet for easy distribution) <i>BLM 9: Rectangle Rodeo</i> <i>BLM 10: Rectangle Rodeo Game Grid</i> <i>BLM 11: Claim the Factors</i> <i>BLM 12: Claim the Factors Product Cards</i> <i>BLM 13: Activity for Home</i> <i>BLM 14.1-2: Learning the Multiplication Facts</i></p>

Timing

2 hours

Preparation and Timing (1 hour and 55 minutes)**Part 1: Getting Started (10 minutes) - with children****Distribute on tables for each group**

Paper, pencils, chart markers, chart paper, scissors, masking tape

Make transparency of:

BLM 1: Welcome

BLM 2: Estimation Instructions

BLM 3: Estimation Question Cards

Part 2: Setting the Stage (30 minutes)**Distribute to each group:**

Tub of square tiles (approx. 40 per person)

Make transparency of:

BLM 4: What is Multiplication?

BLM 5: One-Inch Graph Paper

Make 25 copies for each group

BLM 5: One-Inch Graph Paper

Part 3: Building Arrays (30 minutes)**Prepare:**

Butcher paper, folders, or blackboard with numbers 1-25, spaced for arrays to be posted under each number.
12 items to use for packaging (i.e. egg carton and other packaging) - optional

Part 4: Discussing the Arrays (20 minutes) - without children**Make copies for each participant:**

BLM 6: Finding Multiples

BLM 7: List of Terms

Part 5: Connections (15 minutes)**Make transparency of:**

BLM 8: NCTM Number and Operations Standard

Part 6: Applications for Home (5 minutes)**Distribute to each participant:**

3 index cards, 3x5

Make copies for each participant:

BLM 9: Rectangle Rodeo

BLM 10: Rectangle Rodeo Game Grid

BLM 11: Claim the Factors

BLM 12: Claim the Factors Product Cards

BLM 13: Activity for Home

BLM 14.1-2: Learning the Multiplication Facts

Part 7: Closing (5 minutes) - with children

Distribute evaluations and estimation prizes

Facilitator Resources

Articles

Caliandro, Christine Koller. "Children's Inventions for Multidigit Multiplication and Division." *Teaching Children Mathematics*, February 2000, p. 420.

Kamii, Constance, Barbara A. Lewis, and Sally Jones Livingston. "Primary Arithmetic: Children Inventing Their Own Procedures." *Arithmetic Teacher*. December, 1993. p. 200 -203.

Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, p. 78, ISBN 0-87353-480-8, www.nctm.org

Morrow, Lorna J. (ed). *The Teaching and Learning of Algorithms in School Mathematics (1998 Yearbook)*. National Council of Teachers of Mathematics (NCTM). Reston, VA: 1998. ISBN 0-87353-440-9.

"Whither algorithms? Mathematics educators express their views." p. 1-6

"Children's Invented Algorithms for Multidigit Problems."

"The Harmful Effects of Algorithms in Grades 1-4."

Burns, Marilyn and Tank, Bonnie. *A Collection of Math Lessons, Grade 1-4*. Math Solutions Publications. 1988.

Burns, Marilyn. *Math By All Means: Multiplication, Grade 3*. Math Solutions Publications. 1991.

Ward, Sandra, *Constructing Ideas About Patterns. Grade 1-3*. Creative Publications. 1995.

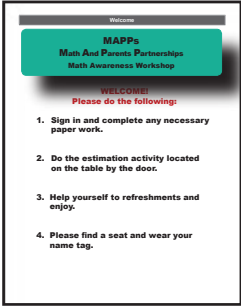
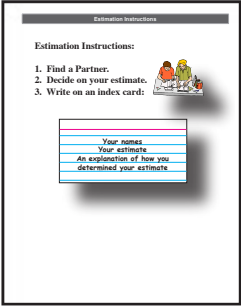
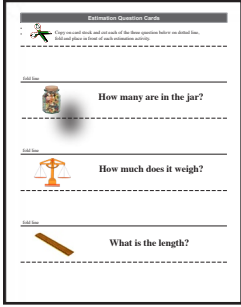

Payne, Joseph N. *Mathematics for the Young Child: Developing Number Sense*. Math Solutions Publications. 1990.

Yates, Jerry and Judy. *Helping Hand: Manipulative Math Handbook of Basic Arithmetic Concepts. Grade 3 and 4*. Advanced Systems. 1990.

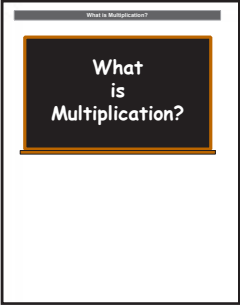

Instructional Programs

Investigations in Number, Data, and Space, TERC, Grade 4: *Packages and Groups (Multiplication and Division)*, Dale Seymour Publications. 1998.

Activities

Preparation of Classroom	Notes
<p>1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. Use handout of BLM 2: Estimation Instructions as instructions for this table (making a transparency of BLM 3 is optional). Prepare BLM 3: Estimation Question Cards and place the cards in front of each estimation activity. These questions are written in generic terms so that they can be used with most estimation questions. Arrange desks or tables in groups of 4-6.</p> <p>2. Display the transparency of BLM 1: Welcome!.</p> <p>3. Distribute pencils, tape, scissors, paper, colored markers, and color tiles on participant tables.</p> <p>4. Prepare an area for posting arrays in front of the room. Write the numbers 1-25 on a blackboard, on folders (one per number), or on butcher paper.</p> <p>5. Around the room, post chart paper that is labeled at the top with "Things that come in groups of 2", another that is labeled "Things that come in groups of 3", and continue with groups of 4 through 12.</p> <p>6. Prepare and display a poster with the agenda and purpose of the session.</p>	<p>BLM 1: Transparency</p>  <p>BLM 2: Transparency / Handout</p>  <p>BLM 3: Handout</p> 
Part 1: Getting Started (10 minutes) - with children	
<p>Charting ideas about groups As participants enter, give them a marker and have them write ideas on the "Things that come in groups" charts. They are to write things that they can think of for each category. See Note A.</p> <p>Introductions</p> <ol style="list-style-type: none"> 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPs program. Have participants who are involved in the program share their experiences. 3. Go over the agenda and purpose for the session. 	<p>A. Note: Example chart</p> 

Activities

Part 2: Setting the Stage (30 minutes)	Notes
<p>Exploring multiplication</p> <ol style="list-style-type: none"> Let participants know that tonight's activities will focus on multiplication so you would like them to think about multiplication. Display the transparency BLM 4: What is Multiplication?. Ask the group: <ul style="list-style-type: none"> <i>What is multiplication?</i> <i>When do we use it?</i> <i>Why do we use it?</i> Give each person a few minutes to write down some ideas. Have groups discuss their ideas. Have groups share while recording their ideas on chart paper. This activity gets participants to record their current thinking about multiplication. It is not intended as a time to teach, but rather, a time to listen. <p>Things that come in groups</p> <ol style="list-style-type: none"> Say: <ul style="list-style-type: none"> <i>Let's look at the things that come in groups that we have around the room. What are some of the things you wrote on the chart?</i> Go over a few examples. Be sure that eggs are listed under groups of 12. Say: <ul style="list-style-type: none"> <i>Let's look at things that come in 2s and make up a story.</i> Give an example of a story problem of things that come in groups. <ul style="list-style-type: none"> <i>"One day I looked out of my window and saw three sets of eyes looking back at me. How many eyes did I see?"</i> Have participants think about how they solved this problem. Have volunteers share their process. Record their responses. See Note B. Make sure the idea of repeated addition is shared. If a participant does not share it, say that one student recorded it as $2 + 2 + 2$. Was he right? What is the connection between addition and multiplication? Ask a volunteer to use the charts to make up a different story. Record the story, draw a picture and record the problem mathematically on a transparency. Ask participants to have their children think of another story that requires multiplication. It should end with a 	<p>BLM 4: Transparency</p>  <p>B. NOTE: Examples of how participants might have solved the eyes problem.</p>  $2 + 2 + 2 = 6$ <p>3 sets of 2 equals 6 3 times 2 equals 6 $3 \times 2 = 6$</p>

Activities

Part 2: Setting the Stage (30 minutes) - continued

Notes

question. Have participants share their story with other at their tables. Encourage them to record the mathematics for their stories.

7. Have participants look at their "What is Multiplication?" ideas that were recorded earlier. Ask if some of the ideas have been discussed. Ask if there is anything that needs to be added to the list. Record the two ideas of multiplication as groups of items and repeated addition on the transparency of "What is Multiplication?."

Introducing arrays

1. In order to lead into the idea of arrays, say to the participants:

We have been looking at multiplication as groups of things and as repeated addition. It is time for us to look at multiplication differently. Let's look at groups of 12.

2. Review the list of things that come in 12. Let them know that the focus will be on eggs. Have an egg carton to verify the packaging.

- *How are a dozen eggs usually packed? (2 rows of 6) We are going to use 12 tiles to represent 12 eggs. Use the tiles to make 2 rows of 6.*
- *How many other ways can you think of packaging a dozen eggs in rows? Use 12 tiles on your table as if they were eggs, and arrange them into rows. Then write down your ideas or draw them on your paper.*

3. Give participants a few minutes to gather their ideas, then ask them how they would describe their cartons. If a 1 by 12 carton is not suggested, ask if anyone thought about a 1 x 12 or 12 x 1 carton (they will be using this shape later).

4. Explain that rectangular arrangements can be called arrays. Write the word array. Mathematically, we can describe a regular egg carton as a two by six (or 6 x 2) array. **See Notes C.** When objects are in an array, we can find the total number of objects if we know the number of rows and how many objects are in each row. For example, when we have 2 rows of 6 eggs we have 6 + 6 or 12 eggs.

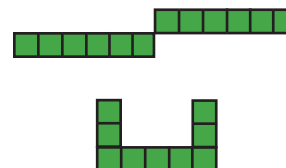
Say:

- *You have just been forming arrays. The number 12 has six arrays: 1 x 12, 12 x 1, 3 x 4, 4 x 3, 2 x 6, and 6 x 2. Draw some examples of these arrays. (Write these on chart paper.)*
- *An array is an arrangement of rows and columns. (You may also want to draw examples that are not arrays.)*

C. NOTE: When participants are designing egg cartons, the idea is to make rectangles or arrays. Below are some examples of arrays:



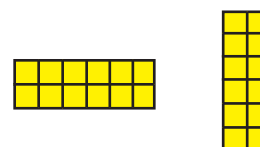
These would not be considered arrays:



Arrays are rectangles. Squares are special rectangles. Therefore, arrays may also be square-shaped.



Here are the two ways to show a dozen eggs packed by twos. One is 6 x 2 and the other is 2 x 6.



Activities

Part 2: Setting the Stage (30 minutes) - continued

- Notice that each size of array may be placed either with the longest side horizontally or vertically. We want to represent the arrays both ways in the project that we are about to start.

Representing arrays (rectangles) on graph paper

1. Use a transparency of **BLM 5: One-Inch Grid Paper** to model an array while saying:

If I were to draw an egg carton on graph paper, it might look like this.

(Draw 2 rows of 6 on the transparency. Draw a large dot in each square to make it easier to count the squares.)

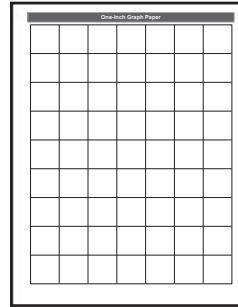
2. Have participants make all the arrays for 12 using graph paper.

3. Have one participant bring up an array and glue or tape it vertically by the number 12 that is posted. Then have another participant put a different array up for 12, and then another until all six arrays for 12 have been posted. (1 x 12, 12 x 1, 2 x 6, 6 x 2, 4 x 3 and 3 x 4). **See Note D.**

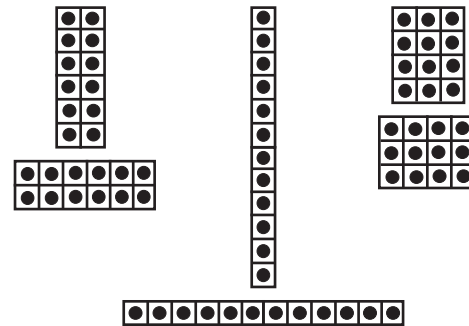
4. Explain that participants are going to build arrays for all of the numbers from 1-25. All of the possibilities for each number will be cut out and pasted by the number.

Notes

BLM 5: Transparency / Handout



D. NOTE: When participants are displaying their arrays, they will display them horizontally and vertically. Having participants draw large dots in each square makes the arrays more visible from a distance. Below are the examples of arrays for 12:



Part 3: Building Array (30 minutes)

Assigning numbers

Assign participants numbers for building arrays. Have the numbers 1-25 on cards and distribute any one or two numbers to partners. **Note E** is for facilitator use only. It shows how many arrays participants should discover for each number. When partners finish their number, they can get a new number.

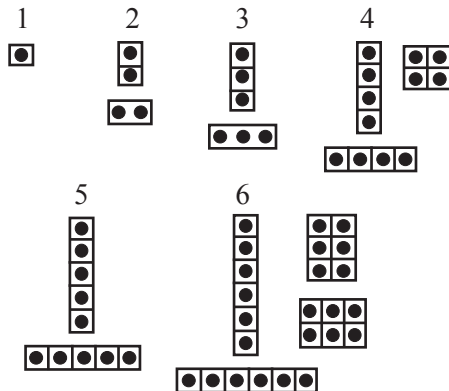
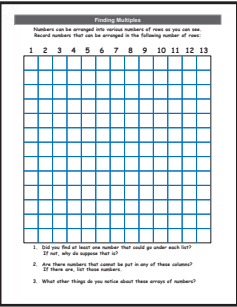
Making arrays

- Have participants begin by building arrays with tiles.
- Distribute the **BLM 5: One-Inch Graph Paper** and have the participants cut out the different rectangles for each of their assigned numbers. Explain the need to tape the grid paper together to form arrays that are longer than 8 units.

E. NOTE: Below are the possible arrays (for facilitator use only). Consider the number of arrays for each number when assigning them to groups.

1: 1x1	15: 15x1, 1x15, 3x5, 5x3
2: 2x1, 1x2	16: 16x1, 16x1, 2x 8, 8x2, 4x 4
3: 3x1, 1x3	17: 17x1, 1x17
4: 4x1, 1x4, 2x2	18: 18x1, 1x18, 2x9, 9x2, 3x6, 6x3
5: 5x1, 1x5	19: 19x1, 1x19
6: 6x1, 1x6, 2x3, 3x2	20: 20x1, 1x20, 4x5, 5x4, 2x10, 10x2
7: 7x1, 1x7	21: 21x1, 1x21, 3x7, 7x3
8: 8x1, 1x8, 2x4, 4x2	22: 22x1, 1x22, 2x11, 11x2
9: 9x1, 1x9, 3x3	23: 23x1, 1x23
10: 10x1, 1x10, 2x5, 5x2	24: 24x1, 24x1, 2x12, 12x2, 3x8, 8x3, 4x6, 6x4
11: 11x1, 11x1	25: 25x1, 1x25, 5x5

Activities

Part 3: Building Array (30 minutes) - continued	Notes
<p>3. Have the participants post these under the designated number on the class chart.</p> <p>4. Remind participants to make a large dot in each square. This allows the rows and columns to be seen and counted when posted. For an example of how these arrays will look, see Note F.</p> <p>5. As participants are cutting out their arrays, remind them that all squares are rectangles, thus they need to cut out the 2x2s, 3x3s, etc.</p>	<p>F. NOTE: The class chart should look similar to this, except that it will have more numbers.</p> 
Part 4: Discussing the Arrays (20 minutes) - without children	
<p>If children are present have them leave at this time in order to allow parents to have an opportunity to learn. Ask participants to combine with another partnership to form a group of 4.</p> <p>Chart observations</p> <p>1. Hand out worksheet BLM 6: Finding Multiples. Model finding numbers that have arrays that come in rows of 2 by listing 4, 6, and 8 under the 2. Have participants take five minutes to look at the class number charts and make observations. Have them record their observations on the worksheet.</p> <p>2. Ask: <i>Did you find examples for each of the numbers?</i> (They should have found some for every number except 13. 13 doubled is 26. Since the arrays only go up to 25, there is no number that comes in rows of 13). Ask the participants why there was no example for 13.</p> <p>3. Introduce "multiples" as a vocabulary word. See Note G.</p> <ol style="list-style-type: none"> Write it on the chart. Explain that they are looking at the multiples of 2 when they look down the list under the 2. Ask them to name some multiples of 10 that are not on their sheet. Ask them to decide on a definition of multiples with their group. Have the groups share their definition, and record one on the chart. 	<p>BLM 6: Handout</p>  <p>G. NOTE: It is not necessary for participants to discover the formal definition of multiple, but it is important that they have a clear understanding of what a multiple is. Multiple: A multiple of a number is the product of that number and any whole number (multiples of 3 are 0, 3, 6, 9, 12, . . .), e.g., all the multiples of 3 have an array that has a length and/or width of 3.</p>

Activities

Part 4: Discussing the Arrays (20 minutes) - continued	Notes
<p>4. Ask:</p> <ul style="list-style-type: none"> • <i>What number did you find that could not be put into any of the columns on your sheet?</i> <p>(Have participants share their lists)</p> <ul style="list-style-type: none"> • <i>What do all of these number have in common?</i> <p>(They only make 2 arrays) These numbers are not multiples of any other numbers. They are call prime numbers.</p> <p>5. Ask:</p> <p><i>What other things did you notice about these arrays of numbers?</i></p> <p>Start by having one group share an observation with the whole group. As they do, record their observations on chart paper and ask them to explain why this pattern occurs. Below is a list of things that might be shared and questions that could be asked:</p> <p>a) Odd numbers are not multiples of even numbers.</p> <ul style="list-style-type: none"> • <i>Why is this so?</i> • <i>What makes an even number?</i> <p>(Have the participants build several even numbers and check to see that they can be arranged in two rows.)</p> <ul style="list-style-type: none"> • <i>What happens if you double the amount?</i> • <i>Triple it?</i> <p>b) Both even and odd numbers are multiples of odd numbers.</p> <ul style="list-style-type: none"> • <i>Why is this so?</i> <p>(Have participants build a row of 3 and then look at what happens to the multiples.)</p> <ul style="list-style-type: none"> • <i>Every other one will by even. Why?</i> <p>c) Most arrays come in pairs: 2×6 and 6×2.</p> <ul style="list-style-type: none"> • <i>Why is this so?</i> <p>(This is a good time to introduce factors as a vocabulary word.)</p> <ul style="list-style-type: none"> • <i>Factors do not come in pairs when the length and width of the array are equal (5×5).</i> <p>d) There are a few numbers that have an odd number of arrays.</p> <ul style="list-style-type: none"> • <i>What do they have in common?</i> <p>(They have an array that makes a perfect square.)</p> <ul style="list-style-type: none"> • <i>Point out the squares formed by the numbers 1, 4, 9, 16, and 25.</i> • <i>Ask them what the next perfect square would be.</i> <p>(This is a good time to reinforce the concept that squares are a special type of rectangles.)</p>	

Activities

Part 4: Discussing the Arrays (20 minutes) - continued

6. Ask:

What do arrays and multiples have to do with the two concepts that we discussed earlier about multiplication?

(Display **BLM 3: What is Multiplication?** with the notes on it from part 2).

Have groups discuss this question and then have them share their ideas. Some of the ideas that participants have shared are:

- a) The rows or columns are like the groups of items.
- b) If you add the rows, it is like repeated addition.
- c) The arrays are a picture of the groups of items or repeated addition.

6. After the groups have shared, tell them that many children learn their multiplication facts by learning to count by two's, three's, or five's. This is often called skip counting.

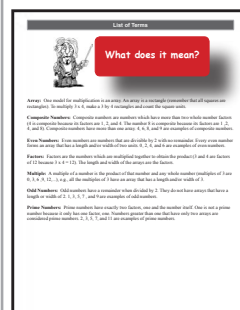
- a) Show participants how to help their children learn to skip count by using skip counting by 3s.
- b) Whisper "1", "2", then loudly say "3". Whisper "4", "5", and then loudly say "6". Continue this for a few more times.
- c) Have participants practice skip counting by 4s in their small groups.

The ability to skip count is a very useful tool for students.

7. Distribute **BLM 7: List of Terms**. Participants may find this to be a helpful reference as they work with their children.

Notes

BLM 7: Handout



Part 5: Connections (15 minutes)

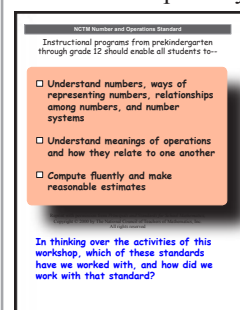
1. Tell participants that we are now going to explore why we looked at all of these ways for thinking about multiplication. Display **BLM 8: NCTM Number and Operations Standard**.

Tell participants that mathematic educators contributed to a guideline for what should be taught in mathematics and how it should be taught. This is a quote from their findings, which is in a book called Principles and Standards for School Mathematics.

2. Have participants think about the experiences that they had in this workshop and decide which of these standards were being used and when they were used.

3 It is important to take time to process the session and help participants connect their experiences to what they already know and to what they would like to do with their children. In order to do this, say:

BLM 8: Transparency



Activities

Part 5: Connections (15 minutes) - continued

Think about how you learned mathematics. How was it like what we did in this session, and how was it different?
(Ask participants to share.)

4. Ask:

How can you use the information from this session to help you help your children learn about multiplication?
(Have participants share their ideas.)

5. If it is not mentioned, remind them that it is helpful to have their children make up story problems about things that come in groups. Both parents and child should make up the stories with imagination and humor. Give an example, such as:

There are 5 people in the family eating dinner and each eats 3 tortillas. How many tortillas need to be cooked?

6. Make sure that participants realize that we are not recommending that they use arrays with their children every time they practice multiplication. The reason for using arrays is for children to have a picture of the meaning of multiplication.

Notes

Part 6: Application for Home (5 minutes)

1. These are games that help children develop an understanding of multiplication and help them learn their facts. Distribute to each participant:

- BLM 9: Rectangle Rodeo**
- BLM 10: Rectangle Rodeo Game Grid**
- BLM 11: Claim the Factors**
- BLM 12: Claim the Factors Product Cards**

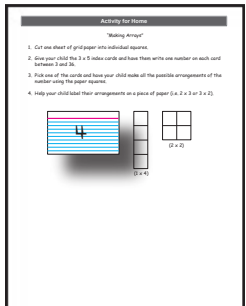
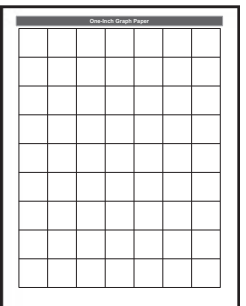
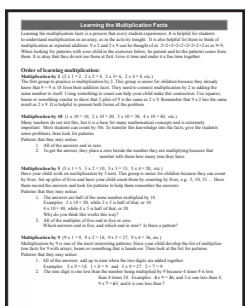
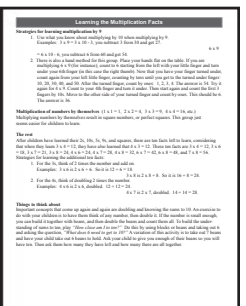
BLM 9: Handout

BLM 10: Handout

BLM 11: Handout

BLM 12: Handout

Activities

Part 6: Application for Home (5 minutes) - continued	Notes
<p>2. Suggest that they make arrays at home with their child/children using the BLM 13 and BLM 5 handouts. Distribute to each participant:</p> <p>BLM 13: Activity for Home BLM 5: One-Inch Grid Paper (4 each) Index cards, 3 x 5 (3 each)</p> <p>3. Included in the take home materials is information on teaching children the multiplication facts.</p> <p>BLM 14.1-2: Learning the Multiplication Facts</p>	<div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>BLM 13: Handout</p>  </div> <div style="width: 50%;"> <p>BLM 5: Handout</p>  </div> <div style="width: 50%;"> <p>BLM 14.1: Handout</p>  </div> <div style="width: 50%;"> <p>BLM 14.2: Handout</p>  </div> </div>
Part 7: Closing (5 minutes) - with children	
<p>1. If your district does not have an evaluation form to use, have them answer one of the following questions:</p> <ul style="list-style-type: none"> • <i>What did you learn tonight?</i> • <i>What will you do with your child as a result of this session?</i> <p>2. Have children rejoin their parents and then distribute any prizes from estimations or drawings.</p>	