Wrap It Up!

Geometry: Netting

Outcomes

- To build an awareness of how student-centered activities and discovery can be used to develop mathematical concepts.
- To provide a basis for mathematical conversations between parents and children.
- To make carreer connections to mathematics.

Overview

In Wrap It Up!, participants explore the nets of cubes. Nets are flat patterns that fold into a three dimensional object. Nets of cubes are a collection of 6 connected squares that, when cut out, fold into a cube. In order to begin this module, the concept of netting is introduced. The definition is shared and participants guess as to which nets on BLM-3 make a cube.

Before participants start building nets, the idea of rotations and reflections is introduced and the participants should come to the conclusion that rotations and reflections of one net are still considered that net.

Next, participants build cubes with their manipulatives, unfold them to find the net, draw the net on the grid paper, and finally cut them out and label them. They should develop 2 piles of cutouts: one pile for those nets that make cubes, and one pile for those nets that do not make cubes. The nets are later taped to the butcher paper that is in the front of the room so that everyone may see them (Refer to the resource page for a picture of the nets). Participants discuss patterns that they see including what makes a net and what will not make a net. At the end of the discussion of the nets, participants fill out a concept map.

Connections are then made with the packaging industry. There are degrees available in packaging engineering. This connection helps participants realize that geometry and measurement are important parts of a well-rounded mathematics background.

As the final processing piece, participants are asked to look at their original guesses as to which nets would make cubes. Participants are asked which ones surprised them and why, and would they be able to guess better now.

Participants are given ideas for working with their students at home. The materials include some netting ideas, some examples of nets, and some web sites for the families to explore.

Note: Polydron frameworks make this module more effective. Parents and students enjoy building with them and are able to visualize the nets better after working with manipulatives.

Mathematics Background

What Experts Say About Spatial Reasoning

The main mathematical focus of this module is spatial reasoning. In Principles and Standards for School Mathematics, the standard of geometry is discussed. It states that students' skills in visualizing and reasoning about spatial relationship are fundamental in geometry. Some students may have difficulty finding the surface area of three-dimensional shapes using two-dimensional representations because they cannot visualize the unseen faces of the shapes. Experience with models of three-dimensional shapes and their two-dimensional nets is useful in such visualization.

Experience with Three-dimensional Objects

As participants experience this module, they may not realize how important it is to connect a solid with its individual pieces. Some students view solids (three-dimensional shapes) as a whole instead of seeing the parts. By manipulating them and looking at the parts, students start to see that solids have two-dimensional faces: sides, fronts, backs and ends. Their explorations can include questions like: What are the shapes and the dimensions of the different faces? Do any of the dimensions match? Why is this so? They can also learn that the sum of the areas of all the faces is the surface area of the three-dimensinal shape.

Connections with Careers

It is interesting that there are college degrees in packaging. The packaging industry has changed remarkably in the last 50 years! Think of the glass bottles used for milk and how they have been replaced by plastics, and even pouches for smaller amounts. The packaging industry is related to netting because most of the time a packaging company needs to design nets for their packages. One challenge is to find a way to lay out the nets so that the least amount of material is wasted. An extension for this problem is to explore the number of nets that can fit on a standard grid sheet. What is the largest number of nets that can fit? How must they be arranged?

Nets of Cubes

When asked to describe the characteristics that make a net of a cube, participants have shared some of the following insights:

- They need to have 6 squares.
- The nets have a total length of at least 4 (one has 5.).
- The width looks like it must be 3 (only one net has a width of 2).
- 4 squares cannot meet in one vertex.
- The perimeter of the net is 14 sides in length.
- There are 24 sides to all the squares that make up the nets:
- There are 5 pairs of sides that are connected in the nets forming edges when they are folded.
- The 5 pairs of sides are made of 10 sides.
- Those 10 sides added to the 14 around the perimeter make 24.
- Each edge of the cube is made of 2 sides touching.
- The 24 sides divided by 2 gives the 12 edges of a cube (Refer to concept map).
- Similarly, vertices are created when 3 sides meet.
- 24 sides divided by 3 gives the 8 vertices (Refer to concept map).

It is important to take enough time to process the things that were learned about cubes by doing this activity.

Room Setup

- Desks or tables arranged in groups of 4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda
- Space in front to hang butcher paper in two colors
- Space to work in partnerships

Materials

Facilitator	Transparencies
• Overhead projector • Overhead pens • Transparencies, write-on • Easel Stand • Chart paper • Chart markers • Masking tape • Timer (optional) • Butcher paper (2 colors) • Scissors • Estimation questions (prepared by facilitator) • Inexpensive prizes	*BLM 1: Welcome* *BLM 77: Are they the same?* *BLM 79: Concept Definition Map* *BLM 81: NCTM Geometry/Representation Standards*
Participant	**Handouts**
Individuals • Paper • Pencil • Reflection • * Polydrons (optional) • Masking Tape • Transparent Tape • Scissors * Polydron Frameworks could also be used. Polydrons cost anywhere from $33 - $60 depending on the size of the set ordered. Squares cannot be ordered separately.	**One per participant for class** *BLM 75: Cube* *BLM 76: Does it make a cube?* *BLM 78: Grid Paper for Nets* *BLM 79: Concept Definition Map* *BLM 80: Career Opportunities* *BLM 81: NCTM Geometry/Representation Standards* **One per participant for home** *BLM 82.1-2: Nets for Home* *BLM 83: Applications for Home* *BLM 84: List of Terms*

Timing

2 hours

Preparation and Timing (1 hour and 55 minutes)

Part 1: Getting Started (10 minutes)

Display transparency from workshop one:
BLM 1: Welcome
Make a copy for each participant:
BLM 75: Cube

Part 2: Setting the Stage (10 minutes)

Make transparencies of:
BLM 77: Are they the same?
Make a copy for each participant:
BLM 76: Does it make a cube?

Part 3: Nets of Cubes (45 minutes)

Make a copy for each participant:
BLM 78: Grid Paper for Nets

Part 4: Processing (25 minutes)

Make transparencies of:
BLM 79: Concept Definition Map
Make a copy for each participant:
BLM 79: Concept Definition Map
BLM 80: Career Opportunities

Part 5: Connections (10 minutes)

Make transparencies of:
BLM 81: NCTM Geometry/Representation Standards
Make a copy for each participant:
BLM 81: NCTM Geometry/Representation Standards

Part 6: Take Home Applications (10 minutes)

Make a copy for each participant:
BLM 82.1-2: Nets for Home
BLM 83: Applications for Home
BLM 84: List of Terms

Part 7: Closing (10 minutes)

No handouts or transparencies
3-4 Inexpensive prizes for Estimation Question winners
Reflection / evaluations *(provided by the evaluation team)*

Facilitator Resources

Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, P. 232 and 280, ISBN 0-87353-480-8, www.nctm.org

Manipulatives:

ETA Math, 1-800-445-5985, www.etauniverse.com
Creative Publications, 1-800-624-0822, www.creativepublications.com

Instructional Programs

Mathematics in Context, Grade7 / 8: *Packages and Polygons* (Sections A and B), Britannica. 1998.

Hatfield, L., *Investigating Mathematics, an Interactive Approach*, (Unit 6: Size and Shape in the Plane), Glencoe, Division of Macmillian/McGraw-Hill Publishing Co. 1994.

Interactive Mathematics Program, Grade 10: *Do Bees Build It Best?* (Netting the Box), Key Curriculum Press. 1998.

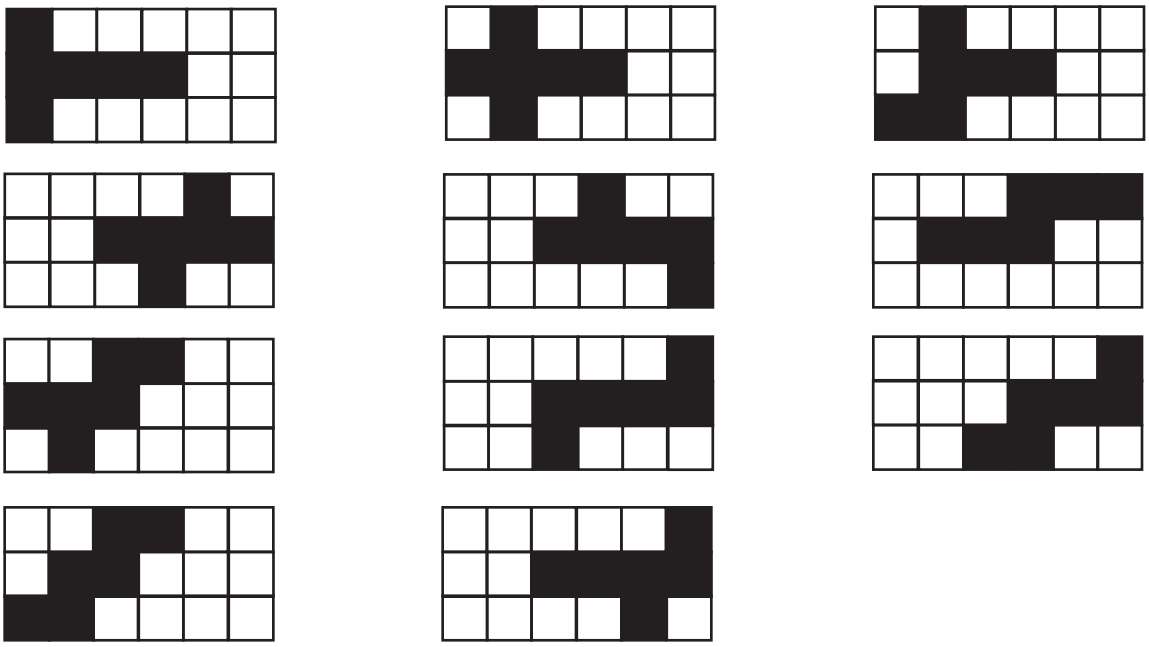
Articles

Lehrer, Richard, *The Shape of Space: Nets*, Mathematics for Parents
www.wcer.wisc.edu/MIMS/Parent_Newsletters/
Lehrer, Richard and Curtis, Carmen L., *Why Are Some Solids Perfect?*, Teaching Children Mathematics, Vol. 6, No. 5, January 2000, P. 324.

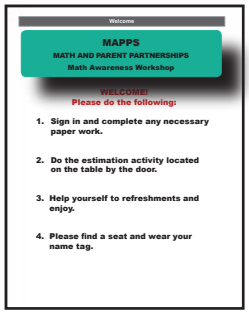
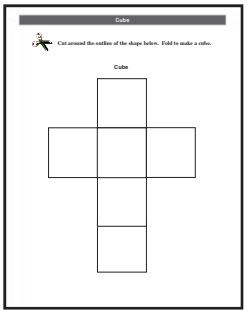
Internet Sites for Parents to Explore

Career Information for Parents: http://stats.bls.gov/k12/html/edu_over.htm
Math, Art, and Fun: http://www.mathartfun.com

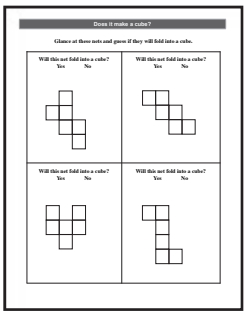
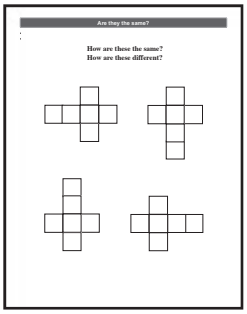
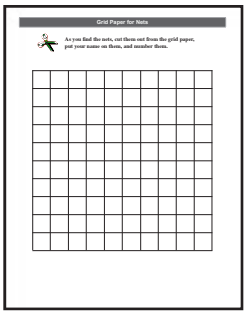
Here are all of the possible nets of a cube that can be discovered. Each of them can be rotated and flipped over to look differently.



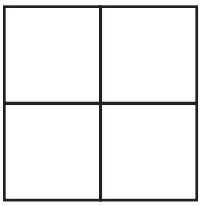
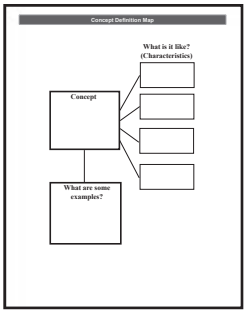
Activities

Preparation of Classroom	Notes
1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up three or four estimation activities. Arrange desks or tables in groups of 4-6. 2. Set up two colors of butcher paper at the front of the room. Label one color "Nets of Cube" for the nets that are able to form cubes (the 11 possibilities are shown in the Facilitator Resources), and label the other color "Not Cubes" for the nets that cannot fold into a cube. 3. Display the transparency of **BLM 1: Welcome!**. 4. Distribute **BLM 75: Cube** to participants tables. If possible, have snap-together manipulatives on each table. **See Note A.** 5. Prepare and display a poster with the agenda and purpose of the session.	BLM 1: Transparency  Welcome MAPPS MATH AND PARENT PARTNERSHIPS Math Awareness Workshop WELCOME! Please do the following: 1. Sign in and complete any necessary paper work. 2. Do the estimation activity located on the table by the door. 3. Help yourself to refreshments and enjoy. 4. Please find a seat and wear your name tag. BLM 75: Handout  Cube Cut around the outline of the shape below. Fold to make a cube. Cube **A. NOTE:** This activity can be enhanced by the use of polydrons (plastic shapes that snap together). They can be used to make the cubes, and then unfold them to find the nets. For ordering information see Facilitator Resources.
Part 1: Getting Started (10 minutes)	
Opening Nets As participants arrive, have them cut about 12 pieces of masking tape 2" to 3" in length, so that they can post their work later. Have them work on the net of a cube, using **BLM 75: Cube**. **Introductions** 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Give participants an overview of the session. Review the agenda and purpose of the session. When discussing the agenda, let the participants know the plan for including children in the session.	

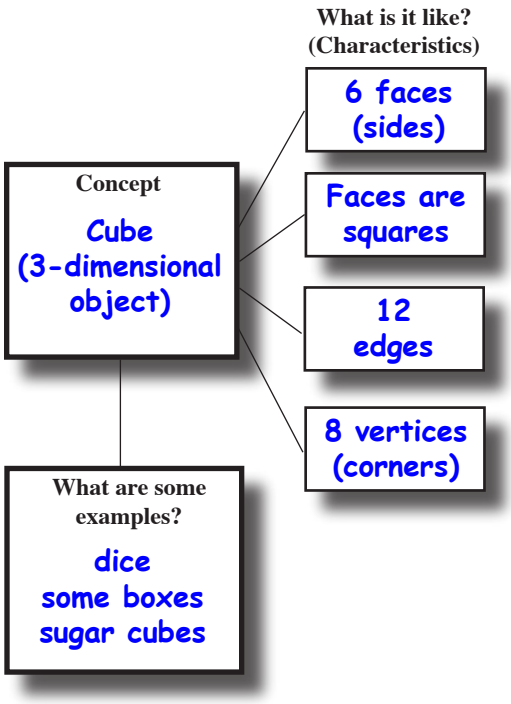
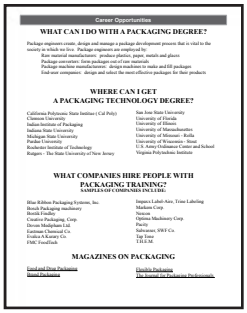
Activities

Part 2: Setting the Stage (10 minutes)	Notes
Introducing Nets of a Cube 1. Have the definition of a net on a poster or transparency. **See Note B.** Tell the participants: *The theme tonight is nets. The definition of a net is a 2-dimensional drawing that folds into a 3-dimensional object. You have just built a cube from a net.* 2. Distribute **BLM 76: Does this make a cube?** to participants. Say: *Look at "Does this make a cube?" and guess which of these will fold into cubes, or in other words, which are nets of cubes?* 3. Have the participants discuss their guesses with their groups. *Put the paper aside. You can check it at the end to see if you have changed your mind about any of them.* 4. Display transparency **BLM 77: Are they the same?.** Ask the participants: • *How are the four nets different?* • *How are they the same?* 5. They should come to the conclusion that they are the same, but rotated (turned) or reflected (flipped). Place the net on the overhead projector and turn it several ways and flip it. State that nets are considered the same if they can be rotated or flipped to match one another. Say: *We call these nets congruent. Your task tonight is to find as many different nets for a cube as you can.*	**B. Note:** The definition of a net is a 2-dimensional drawing that folds into a 3-dimensional object. BLM 76: Handout  Does it make a cube? Glance at these nets and guess if they will fold into a cube. Will this net fold into a cube? Yes No Will this net fold into a cube? Yes No Will this net fold into a cube? Yes No Will this net fold into a cube? Yes No BLM 77: Transparency  Are they the same? How are these the same? How are these different?
Part 3: Nets of Cubes (45 minutes)	
1. Distribute handout from **BLM 78: Grid Paper for Nets.** Ask participants to: *a) Draw various nets on your grid paper that you think might fold into a cube.* *b) Cut each one out, put your name on it.* *c) Fold it to see if it makes a cube.* *d) Keep all nets, even if they don't fold into a cube.* *e) Sort your nets into 2 piles, nets that make a cube; nets that don't make a cube.*	BLM 78: Handout  Grid Paper for Nets As you find the nets, cut them out from the grid paper, put your name on them, and number them.

Activities

Part 3: Nets of Cubes (continued)	Notes
2. Give participants about 20 minutes to work on these before you interrupt them. Find someone that has the net that looks like an upper case " T". Ask them to describe the net that they have. Then have them tape it to the butcher paper for "Nets of Cubes". Ask each group to display a net that has not yet been shared. Continue in this fashion until several types of nets have been displayed. 3. Have participants match their nets to the ones that are posted and tape them underneath. Give participants enough time to compare their nets to the others that are displayed. After awhile, ask everyone to tape up the nets that didn't make a cube on the poster "Not Cubes". Ask: *What observations did you make about nets which do/don't make cubes?* 4. Ideas that could be shared are: Cube nets all have 6 squares, and it doesn't work to have 4 squares meeting at one vertex. **See Note C**. After a total of 40 minutes, ask everyone to post new nets they have found. Ask: *Does anyone have any new observations they could share?* Have participants share their ideas. 5. Have students leave at this point so that parents will be able to focus and reflect on the mathematics.	**C. NOTE:** Here is an example of 4 squares with one vertex. The one vertex is in the center of the 4 squares. 
Part 4: Processing (25 minutes) - no children	
1. Distribute **BLM 79: Concept Definition Map** shown on next page. **See Note D** on next page. Start by saying: *Let's take a moment to discuss the characteristics of a cube. One of the ways to explore ideas is to use a concept definition map.* 2. Display the transparency of **BLM 79: Concept Definition Map.** Say: *Since our concept for this map is cubes. We will write cube for concept.* (Write cube in the concept rectangle on the transparency.) Then ask: *What characteristics of cubes can you name?* (Write in the characteristics on as participants share.) 3. After they have generated a list of characteristics, ask: *What are some examples of cubes around our daily lives?* (Write them in as participants share.)	BLM 79: Transparency / Handout 

Activities

Part 4: Processing (continued) - no children	Notes
Using Nets 1. Possible questions: *What are some everyday examples of nets people use?* (ie. patterns in sewing, making boxes, wrapping gifts, model cars, and model airplanes, are some ideas). Nets are used in the packaging industry. 2. Distribute **BLM 80: Career Opportunities** and discuss the opportunities for a career in packaging technology. The discussion can include anything that surprises participants. Some ideas are: a) The US Army offers training in this field. b) There are monthly magazines exclusively for packaging. c) Universities offer degrees in packaging technology. 3. Tell the participants: *Mathematics has many different branches: algebra, geometry, data analysis, and probability for example. Each one has a different emphasis and takes different talents. Each one can lead to a career that utilizes these talents. It is important to help your child determine where their talents are and select careers that match their talents. Information on careers can often be found at the school's counseling center, a community college, or at web sites. A web site from the Department of Labor is included in your handouts later tonight.* 4. Businesses are always looking for ways to cut the cost of materials. An extension question for tonight's activity is that a packaging business would ask is: • *What if your grid sheet tonight was really made of metal and cost $10.00 per grid and a company is willing to pay you $2.00 for every cube you can make out of it.* • *How many nets can you fit on the grid.* • *How much money will you make?* • *Can you find a way to fit more on?* • *Why is the amount of waste important to companies?* (The profit is greater when there is less waste.) 5. Say: *Get out your sheet of "Does it make a cube?" to see how you did. Would you change you mind about any of your answers?* 6. Give participants a few minutes to discuss this with their group. Then ask for volunteers to share.	**D. NOTE:** Concept Definition Map **Characteristics of a Cube**  BLM 80: Handout  Career Opportunities WHAT CAN I DO WITH A PACKAGING DEGREE? WHERE CAN I GET A PACKAGING TECHNOLOGY DEGREE? WHAT COMPANIES HIRE PEOPLE WITH PACKAGING TRAINING? MAGAZINES ON PACKAGING

Activities

Part 5: Connections (10 minutes)	
1. Ask: *What mathematics did you do tonight?* Have participants discuss this question in their groups and then share their thoughts. Answers will probably include: a) Mathematics vocabulary b) Geometry c) Surface area **2.** Connect the nets activity to the standards that students need to be learning. Do this by connecting it to your state or local standards (ask your district for these), or the National Standards (display the transparency of **BLM 81: NCTM Geometry/Representation Standards**). Say: *There are 10 mathematical standards. They are number and operations, algebra, geometry, measurement, data analysis and probability, problem solving, reasoning and proof, communication, connections, and representations. Your experience tonight has touched on many of these. Let's focus our discussion on the geometry and the representation standards.* 3. Distribute **BLM 81: NCTM Geometry/Representation Standards** to the groups and ask them to identify parts that they experienced in tonight's workshop. 4. Have participants discuss the standards in their groups and mark the ones that they experienced tonight (Only the first, third and fourth in Geometry and the first one in Representation should be marked). 5. Have participants report out. Ask participants: • *How is this mathematics different from the way you were taught mathematics?* • *Why do you think the changes were made?* (The introduction of technology and the changes in our world are so rapid that we need to develop problem solvers who will be able to keep up with the changes). 6. Select some examples from your district's curriculum that show how some of the geometry concepts are taught. (See suggestions in facilitator references section of "Preparing for the module").	BLM 81: Transparency / Handout  NCTM Geometry/Representation Standards **Geometry** Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships Specify location and describe spatial relationships using coordinate geometry (graphing) and other representational systems (examples: navigation longitude and latitude, polar coordinates) Apply transformations (flips and rotations) and use symmetry to analyze mathematical situations Use visualization (seeing in your mind), spatial reasoning, and geometric modeling to solve problems **Representation** Create and use representations (models, graphs) to organize, record, and communicate mathematical ideas Select, apply and translate among mathematical representations (example: connect a graph with its equation) to solve problems Use representations to model and interpret physical, social, and mathematical phenomena (example: graph, diagram)

Activities

Part 6: Take Home Applications (10 minutes)

1. Tell participants:

   Imagine what the nets look like for things around your home. Have a game to see how close you can get to making the nets for various objects around your home. For example:

   a) What things were made from a flat sheet of metal?

   b) What things were made from a flat piece of cardboard?

   Tear open the containers to see what their nets look like.

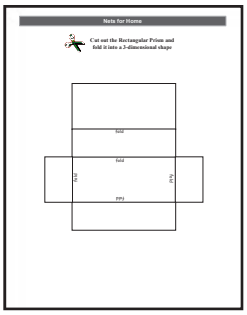
   (It would be helpful here to have a cereal box. Show how it was made by tearing it along glued joints.)

2. Distribute the following handouts:

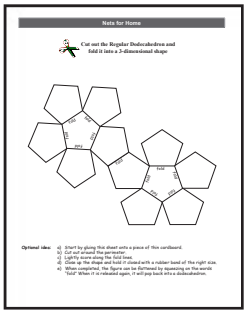
   BLM 82.1-2: Nets for Home
   BLM 83: Applications for Home
   BLM 84: List of Terms

3. Discuss the handouts. **Nets for Home** is an example of some nets to cut out and fold at home. **Applications for Home** includes some of the ideas that were just discussed about nets. It also includes 2 web sites. One is for parents and children to use when exploring careers. The other is one example of a site with fun math activities.

BLM 82.1: Handout

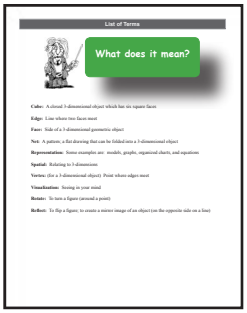


BLM 82.2: Handout



BLM 83: Handout

BLM 84: Handout



Part 7: Closing (10 minutes)

1. Have the participants celebrate the fact that they were here with their children (high 5's, pat each other on the back, applaud themselves, or congratulate each other).

2. Distribute any prizes for Estimation Question winners.

3. If your district does not have an evaluation form to use, you may want to use a reflection similar to:
   - *One thing that surprised me was . . .*
   - *One thing that confused me was . . .*
   - *One thing that pleased me was . . .*

4. Thank participants for coming during their busy schedules.