## Show Me the Money




## Compounded Interest on Spreadsheets

## Outcomes

- To explore compound interest of money.
- To explore the power of computer spreadsheets.
- To experience constructive learning.


## Overview

Show Me the Money, is a technology workshop. It is designed to be conducted in a computer lab using a spreadsheet. It is reccommended that the facilitators be familiar with spreadsheets and how they are set up. It is also important that the facilitator try the activities of the workshop ahead of time on the computers that will be used. This will go a long way toward preventing some of the problems that can arise in a computer lab (like an access code, setting up an overhead screen, opening the spreadsheet, and being alert to computers that are out of order)

This module is intended to engage participants in a constructivist activity using technology. Many parents believe that using technology is a way of cheating. The module is an opportunity to show what a powerful exploration and learning tool computers (and calculators) can be. Participants have the opportunity to write equations in the cells of spreadsheets and do an exploration that recreates Albert Einstein's Rule of 72 , a rule of thumb used to estimate the amount of time an investment takes to double when interest is compounded annually.

At the start of the workshop, participants work with the graphing calculators or with the calculators on the computers to compute the growth of money on the sheet: Growing Your Money. This activity paths the way for the computer activity.

Next, participants are given a basic introduction to spreadsheets. They soon become familiar with the format of a spreadsheet, how to identify cells, and how to write formulas for cells. Using the spreadsheet, they compute the growth of an investment of $\$ 1000$ at $5 \%$ annually compounded, which is the same problem that participants did on their calculator. After successfully setting up the computer to do the problem, participants explore other percentage rates. They develop a table of the rate and the length of time that it takes for the money to double. The table is used for exploring the pattern, by looking at what happens when you multiply the two pieces of information. The activity helps participants discover the rule of 72 for themselves. Einstein had found that the product was close to 72 . Participants look at the rule and try it out with a few percentages so that they can make sense of it.

Time is given for exploring the connection between how the session was taught and the constructivist approach to teaching and learning. The constructivist emphasis in this workshop helps participants understand the difference between being told formulas and constructing them with understanding.

The extension problem is Baby Jedidiah's Fund, which explores the growth of a college fund. Participants need to decide how much money they need to deposit today so the the account will have exactly $\$ 100,000$ in 18 years at $8.25 \%$ interest, compounded annually. They use a spreadsheet in order to guess the beginning amount and then check it on the spreadsheet. They revise their guess, and check again. This activity is enjoyable, engaging, and helps participants see what a powerful tool a spreadsheet can be for such investigations.

The session closes with prizes for the estimation and a written reflection or evaluation.

## Mathematics Background

The mathematical concept in this module is patterns. Participants spend the evening using technology to discover the pattern for the growth of money. It is meant to be an experience with constructivist teaching, having the participants construct the rule of 72 as Einstein did so long ago.

## Calculators

The first activity uses a calculator to do the simple computations needed to complete Growing Your Money. The calculator does not have to be fancy, but you need to know how to direct the participants to use the repeating function. To do this, enter 1000 * 1.05 and press enter. Then press * 1.05 and enter. This should take the last answer and multiply it by 1.05 . Now if you press enter again, the calculator automatically multiplies the most recent answer by 1.05. 1.05 represents the old number (1.00), and the computed growth from interest (.05), combining them into one step of $\times 1.05$. It could be something that is discussed when you discuss converting percents to decimals.

## Annual compound interes $\dagger$

The formula for annual compound interest is $Y=P(1+r)^{x} . V=$ the new amount. $P$ is the original amount, $r$ is the interest rate as a decimal, and $x$ is the number of years.

## Computers

Spreadsheets are very powerful tools that are available to students today. The uses of the spreadsheet are only limited by our imagination as teachers. The exercise in this module is using them to compute interest and then investigating how long it takes money to double. The steps for doing this are laid out in great detail.

## Finding a Pattern

Participants create a chart of the interest rates and the length of time it will take their money to double. Below is a sample of how this table should look.

| Interest Rate | Years to Double | Interest $x$ Years |
| :---: | :---: | :---: |
| 4\% | 18 | 72 |
| 5\% | 14.5 | 72.5 |
| 6\% | 12 | 72 |
| 7\% | 10.5 | 73.5 |
| 8\% | 9 | 72 |
| 9\% | 8 | 72 |
| 10\% | 7.3 | 73 |
| 12\% | 6 | 72 |
| 15\% | 5 | 75 |
| 18\% | 4 | 72 |

## The Rule of 72

Studying the table can lead to the rule of 72 . The rule of 72 states that if you divide 72 by an interest rate, you will estimate the number of years for doubling your money at that rate. Albert Einstein is attributed to discovering this pattern. So, what have the participants done? They have reconstructed a rule of finance, or mathematics. They have made connections with the real world and constructed understanding of the concept, so that the rule of 72 is not a magical formula, but an understandable concept based on patterns. One of the Algebra focuses of the National Council of Teachers of Mathematics is the exploration of patterns. Most of what we know about mathematics has been discovered through finding patterns.

## Room Setup

- Computer Lab (with Excel Application on computers)
- LCD Projector
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda

This module takes place in a computer lab. Have enough computers for the participants. You will need a display mechanism (LCD projector) that shows the computer screen while you are explaining the steps. Also, there should be a banner or chart that has the focus question of the module: How long does it take money to double? A chart of the table: How long to double? could be displayed on an overhead so that it can be filled in with the calculations that participants share.

## Materials

## Facilitator

- Overhead projector
- Overhead pens
- Transparencies, write-on
- Easel Stand
- Chart paper
- Chart markers
- Masking tape
- Timer (optional)
- Butcher paper (optional)
- Calculators
- Computer
- LCD projector (optional)
- Estimation questions (prepared by facilitator)
- Inexpensive prizes


## Participant

## Transparencies

BLM 1: Welcome
BLM 86: Constructivism
BLM 87: NCTM Technology Principle
BLM 91: How long to double?
BLM 92: The Rule of 72

## Individuals

- Paper
- Pencil
- Reflection
- Computer station


## Handouts

## One per participant for class

BLM 85: Growing Your Money
BLM 88: Sample Excel Spreadsheet
BLM 89.1-3: Spreadsheet Instructions
BLM 90: "Growing Your Money" Spreadsheet
BLM 91: How long to double?
BLM 92: The Rule of 72
BLM 93: Baby Jebidiah's Fund
BLM 94: Technology in the Classroom
One per participant for home
BLM 95: Using Technology

## Timing

## Preparation and Timing (2 hour and 20 minutes)

## Part 1: Getting Started (10 minutes)

Display transparency from workshop one:
BLM 1: Welcome
Make transparencies of:
BLM 86: Constructivism
BLM 87: NCTM Technology Principle
Make a copy for each participant:
BLM 85: Growing Your Money
Part 2: Setting the Stage (10 minutes)
No handouts or transparencies
Part 3: Computer Spreadsheet (60 minutes)
Make a copy for each participant:
BLM 88: Sample Excel Spreadsheet
BLM 89.1-3: Spreadsheet Instructions
BLM 90: "Growing Your Money" Spreadsheet
Part 4: How Long to double? (20 minutes)

## Make transparencies of:

BLM 91: How long to double?
Make a copy for each participant:
BLM 91: How long to double?
Part 5: The Rule of 72 ( 10 minutes)

## Make transparencies of: <br> BLM 86: Constructivism <br> BLM 92: The Rule of 72 <br> Make a copy for each participant: <br> BLM 92: The Rule of 72 <br> Part 6: Baby Jebidiah's Fund (10 minutes) - Optional

Make a copy for each participant:
BLM 93: Baby Jebidiah's Fund

## Part 7: Connections (10 minutes)

## Make a copy for each participant:

BLM 94: Technology in the Classroom
Part 8: Take Home Applications (5 minutes)

## Make a copy for each participant:

BLM 95: Using Technology
Part 9: Closing (5 minutes)
No handouts or transparencies
3-4 Inexpensive prizes for Estimation Question winners
Reflection / evaluations (provided by the evaluation team)

## Facilitator Resources

## Articles

Kaput, J. J., \& Thompson. P. W. (1994) Technology in mathematics education research: The first 25 years in the JRME. Journal for Research in Mathematics Education, 25, p. 676-684.

Thomas, E. J. A study of the effects of a computer graphics problem-solving activity on student achievement, attitudes, and task motivation. (Georgia State University, 1989). Dissertation Abstracts International, 51, p. 102A.

## Books

Brooks, Jacqueline Grennone and Martin G.. In Search of Understanding: The Case for Constructivist Classrooms. Association for Supervision and Curriculum Development (ASCD). 1993.

Davis, Robert B., Maher, Carolyn A., and Noddings, Nel. Constructivist Views on the Teaching and Learning of Mathematics. National Council of Teachers of Mathematics (NCTM). 1990.

Davis, Robert B., Maher, Carolyn A., and Noddings, Nel. Constructivist Views on the Teaching and Learning of Mathematics. National Council of Teachers of Mathematics (NCTM). 1990.

Standards 2000 Project, Principles and Standards for School Mathematics, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, p. 24, ISBN 0-87353-480-8, www.nctm.org

## Internet Sites for Parents to Explore

Site addresses updated 9-29-03

Translations: www.systransoft.com/

Homework help: www,mathforum.org/dr.math/
Family Mathematics: www.figurethis.org/

Maps: maps.yahoo.com/

Email free of charge: mail.yahoo.com

## Activities

## Preparation of Classroom

Notes

1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up three or four estimation activities.
2. This module takes place in a computer lab. Have enough computers for each participant or for partners to share. It is helpful to have several student volunteers available to help parents.
3. Set up an LCD projector (display mechanism that shows the computer screen) and a computer with the Excel program.
4. On a whiteboard or chart paper, display the focus question of this session: How long does it take money to double?
5. Display the transparency of BLM 1: Welcome!.
6. Prepare and display a poster with the agenda and purpose of the session.
7. Distribute the handout of BLM 85: Growing Your Money, calculators, and pencils to participants as they arrive.

## Part 1: Getting Started (10 minutes)

## Introductions

1. Introduce yourselves and then have the participants introduce themselves.
2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences.
3. Give participants an overview of the session. Review the agenda and purpose of the session.
4. Display BLM 86: Constructivism. Explain that one approach to mathematical concepts is to allow students to construct their own knowledge. During this workshop, participants will be experiencing learning in this manner.
5. Display BLM 87: NCTM Technology Principle. Read it aloud to the participants and tell them that this workshop will be an experience in using technology to explore mathematics.

BLM 1: Transparency


BLM 85: Handout

BLM 86 : Transparency
/ Handout

## Activities

## Part 1: Getting Started (continued)

6. Have the volunteer students introduce themselves and their parents if they are present: Say:

These volunteer students will be tutors tonight as we explore technology. To ask for their help, raise your hand.

## Growing Your Money

Give the parents an overview of the workshop. Tell them that they will be doing the following:
a. Working on the Growing Your Money worksheet that they have at their tables;
b. Entering the same worksheet on a computer spreadsheet so that the computer will do the work for them;
c. Using the worksheet and the computer to look for patterns about doubling money;
d. Looking at how the evening's activities relate to the mathematics classroom of today.

## Part 2: Setting the Stage (10 minutes)

1. When participants have checked in and worked on a few problems from the Growing Your Money worksheet, bring the group together. See Note A for an expanded version of Growing Your Money.
2. Have them discuss with their neighbor any observations. Ask:

What observations do you have about this activity?
3. Record their responses. Below are some responses you can expect:
a. Amount of money in the account is growing by different amounts each year.
b. Money is growing by larger amounts each year.
c. 05 represents the interest rate of $5 \%$ in decimal form
This last response is an important concept, because it is the conversion from percent to decimals. Take a moment to make sure that participants remember this process. If they have not brought it up themselves, ask:

- What does 05 represent?
- How do you write $8 \%$ ?
- How do you write $12 \%$
- How long would it take for this money to double or become $\$ 2000$ ?
(This question is the focus of this session. It should be posted.)
- To help us answer this question we will use a spreadsheet on the computer.
A. NOTE: Below is a printout of a spreadsheet of Growing Your Money for reference. The amounts may vary slightly depending on rounding numbers off.

| 1 | $\$ 1,000.00$ | 0.05 | $\$ 50.00$ | $\$ 1,050.00$ |
| ---: | ---: | ---: | ---: | ---: |
| 2 | $\$ 1,050.00$ | 0.05 | $\$ 52.50$ | $\$ 1,102.50$ |
| 3 | $\$ 1,102.50$ | 0.05 | $\$ 55.13$ | $\$ 1,157.63$ |
| 4 | $\$ 1,157.63$ | 0.05 | $\$ 57.88$ | $\$ 1,215.51$ |
| 5 | $\$ 1,215.51$ | 0.05 | $\$ 60.78$ | $\$ 1,276.28$ |
| 6 | $\$ 1,276.28$ | 0.05 | $\$ 63.81$ | $\$ 1,340.01$ |
| 7 | $\$ 1,340.01$ | 0.05 | $\$ 67.01$ | $\$ 1,407.10$ |
| 8 | $\$ 1,407.10$ | 0.05 | $\$ 70.36$ | $\$ 1,477.46$ |
| 9 | $\$ 1,477.46$ | 0.05 | $\$ 73.87$ | $\$ 1,551.33$ |
| 10 | $\$ 1,551.33$ | 0.05 | $\$ 77.57$ | $\$ 1,628.89$ |
| 11 | $\$ 1,628.89$ | 0.05 | $\$ 81.44$ | $\$ 1,710.34$ |
| 12 | $\$ 1,710.34$ | 0.05 | $\$ 85.52$ | $\$ 1,795.86$ |
| 13 | $\$ 1,795.86$ | 0.05 | $\$ 89.79$ | $\$ 1,885.65$ |
| 14 | $\$ 1,885.65$ | 0.05 | $\$ 94.28$ | $\$ 1,979.93$ |
| 15 | $\$ 1,979.93$ | 0.05 | $\$ 99.00$ | $\$ 2,078.93$ |
| 16 | $\$ 2,078.93$ | 0.05 | $\$ 103.95$ | $\$ 2,182.87$ |
| 17 | $\$ 2,182.87$ | 0.05 | $\$ 109.14$ | $\$ 2,292.02$ |
| 18 | $\$ 2,292.02$ | 0.05 | $\$ 114.60$ | $\$ 2,406.62$ |
| 19 | $\$ 2,406.62$ | 0.05 | $\$ 120.33$ | $\$ 2,526.95$ |
| 20 | $\$ 2,526.95$ | 0.05 | $\$ 126.35$ | $\$ 2,653.30$ |

## Activities

## Part 3: Computer Spreadsheets (60 minutes)

Notes

## Spreadsheet Basics

1. Determine the participants familiarity with the spreadsheet. Say:

Those who have used computer spreadsheets before, raise your hand.
(This will help you find out how much time to spend on teaching the basics of spreadsheets.)
2. Let participants know that spreadsheets are used routinely in today's world. They help us look at data and perform operations with data quite rapidly.
3. Lead the participants through some basic functions with spreadsheets. Have participants open the Excel program. Make sure everyone is at this point before you proceed. Use an LCD projector with an Excel spreadsheet on the computer to introduce the participants to the idea of cells. Distribute the handout of BLM 88: Sample Excel Spreadsheet for their information. Say:

In a spreadsheet each cell (box) has an identity. On the top you will see $A, B, C$, etc. These are the letters of the columns. Look for the numbers 1, 2, 3, 4, etc. on the side. These are the numbers of the rows. If I refer to $A 3$, it will be in the column $A$ and the row 3. Check with your neighbor to see that you both understand this.
a) Use the mouse to move the arrow (cursor) to cell F4.
b) Click on cell F4.
c) Type your name on the keyboard.
d) Find your name in two places on the screen.
(Have participants check with neighbors to see that their name is in cell F4.)

You should see your name in cell F4 and in the formula bar above your spreadsheet.
e) Press return to tell the computer that you want your name in cell F4.
(Have participants practice entering numbers in various cells.)
f) Save what you have done by pulling down FILE in the top left corner of your screen and choose Save.
g) You will want to name the spreadsheet, "Money". Then press return.
We will ask you to save often, and after this time you will not need to name the spreadsheet.

BLM 88: Handout


## Activities

## Part 3: Computer Spreadsheets (continued)

Notes

## Creating a new spreadsheet

Explain how to open a new computer spreadsheet. (Our example will use Excel.)

In order to open a new spreadsheet in Excel:
a) Choose Excel.
b) Pull down the FILE from the menu bar.
c) Choose New.
d) Click $\boldsymbol{O K}$ for new workbook (or double click on the workbook icon). A new spreadsheet will appear.

## Setting up the spreadsheet

1. Before handing out BLM 89.1-3: Spreadsheet

Instructions have the participants follow your oral instructions for numbering. These instructions are for Excel. Adjust them if you will be using a different program.
a) Click on cell A1.
b) Type the number 1. (You may notice there are two 1's on the keyboard. Either will work).
c) Press return to tell the computer that you want 1 in the cell A1. Now you should have A2 outlined.
d) Type the number 2 and press return. Continue numbering the cells to number 20.
e) Save what you have done by pulling down FILE in the top left corner of your screen and choose Save.
2. While participants number the years 1-20, distribute all pages of BLM 89.1-3: Spreadsheet Instructions. After everyone has the instructions, say:

You have the instructions explaining how to have the computer do the work for Growing Your Money. We will be moving slowly through the instructions on the overhead screen. If you need help please raise your hand and a student volunteer will get to you quickly. We have completed steps 1 through 2 a on your spreadsheet instructions. Start with step $2 b$.
3. The Spreadsheet Instructions are used for step-bystep instructions as you, or a student volunteer, model the procedure on the projected computer image. It is often easier to have the whole group learn the basics of the computer together. Allow participants to move at their own pace by saying:

If you are familiar with computers and want to move at your own pace, please do so.

## BLM 89.1: Handout



BLM 89.3: Handout


## Activities

## Part 3: Computer Spreadsheets (continued)

## Notes

4. Demonstrate the spreadsheet instructions on the overhead slowly and carefully for those who need it. Below are important things to remember as you model the use of a spreadsheet.

## Dialogue box (format cell)

Leave the dialogue box on the screen with currency highlighted.

## Formula in D1

Type the formula in D1 and DO NOT press return so it remains visible in the formula bar.
(One common mistake here is for participants to omit the $=$. To check have the participant click on cell D1 to see the formula.)
Formula in E1
Type the formula in E1 and DO NOT press return so it remains visible in the formula bar.
(One common mistake here is for participants to omit the $=$. To check have the participant click on cell E1 to see the formula.)
Formula in B2
Type the formula in $\mathbf{B 2}$ and DO NOT press return so it remains visible in the formula bar.
(One common mistake here is for participants to omit the =. To check have the participant click on cell B2 to see the formula.)
Fill Down (Pause at each of the three fill downs)
Stop at the highlight. DO NOT do Fill Down.
(One common mistake is to begin in cell B1 instead of B2. We need $B 2$ because that cell contains the formula. If the participants used B1, they will have $\$ 1000$ in each cell. If they fill from cell B2, there will be a number in B2 and $\$ 0.00$ in other cells.)
5. Ask participants:

- What are we doing in column C? (computing 5\%)
- What amount is showing in column D? (the dollar amount for 5\%)
- What did the computer do to get column E? (add B and D, or the original amount plus $5 \%$ )

6. Distribute the handout of BLM 90: "Growing Your

Money" Spreadsheet so the participants know what their spreadsheet should look like. Look at BLM 90 and compare the process. The calculator is like a mini-computer in some ways.


## Activities

## Part 4: How Long to Double? (20 minutes)

1. Distribute the handout of BLM 91: How long to double? to each participant as they finish the spreadsheet instructions.
2. When everyone has completed the spreadsheet, get everyone's attention and say:

You have done an excellent job. Before we did the spreadsheet, we asked the question, how long will it take your money to double, or become $\$ 2000$ ? What is the answer to this question at $5 \%$ ? (about 14.5 years)
3. Have participants look at the spreadsheet that is on their screen. (This will vary because some participants have worked ahead). Say:

Look on your handout, How Long to Double. You will notice that the doubling time for $5 \%$ is already written on it. Use this handout to experiment with other percentages for the next few minutes.
4. Facilitator looks for participants who have confidence and have completed the first task on "How Long to Double" successfully. Ask them quietly to figure $15 \%$ and $18 \%$. Make a chart or display the transparency of BLM 91: How long to double to chart their answers.

## Part 5: The Rule of 72 ( 10 minutes)

1. Ask the participants the following question from the handout of How long to double?:

What number would best represent all of the numbers in the far right column?

The number should be range in the 70's: from 70 to 75 at most. Say:

Since 72 can be divided evenly by so many numbers, financial advisors often use what they call the rule of 72 .
2. Distribute BLM 92: The Rule of 72 and display the transparency of the handout. Go through the ideas on the transparency and have the participants apply to rule to $12 \%$.
3. Display the transparency of BLM 86: Constructivism. Point out that what the participants have been doing is constructivist learning. Ask them how this is different from how they learned and give them a chance to share.

## Notes




## Activities

## Part 6: Baby Jebidiah's Fund (20 minutes) - optional

1. Distribute the handout of BLM 93: Baby Jebidiah's Fund. Have participants guess the amount that needs to be deposited and share it with a neighbor. Then have them check their guess on the computer. Give them time to complete the investigation, then share ideas on their conclusions.
2. Ask:

How could you use the Rule of 72 to help you estimate in this problem?
(If the interest rate is $8 \%$, then the money should double about every 9 years $-72 \div 8=9$. So in 9 years your money will double and in 9 more it will double again. \$25,000 doubled is $\$ 50,000$. $\$ 50,000$ doubled is $\$ 100,000$. So the amount you are looking for must be close to $\$ 25,000$.)

## Part 7: Connections (5 minutes)

1. Revisit the question How Long does it Take Money to Double? Ask participants what they know about this question that they did not know when they came in tonight. After sharing on that idea, ask what mathematics they did in this activity. Some ideas are:
a) Wrote equations in the cells
b) Exponential growth
c) Basic operations on numbers
d) Mathematical logic and reasoning
e) Converting percents to decimals

Ask:
What role should technology play in education today? Have them share their ideas with someone sitting near them, then ask the whole group to share.
2. Distribute BLM 94: Technology in the Classroom. Let them know that mathematics and business leaders have put a lot of thought into this question and have written a statement about it. This paper is summarized from the Principles and Standards for School Mathematics. It explains the national stand on using technology for learning. Tonight had two purposes:
a) For participants to experience constructivism and understand what it means to mathematics education.
b) For participants to think about the use of technology in mathematics instruction.

BLM 94: Handout


## Activities

## Part 8: Take Home Applications (5 minutes)

| 1. Distribute BLM 95: Using Technology. Explain that |  |
| :--- | :--- |
| computers are available at libraries and can be used to |  |
| expand student's experiences. |  |

1. Distribute any prizes for Estimation Question winners.
2. If your district does not have an evaluation form to use, you may want to use a reflection similar to:

- What did you learn tonight?
- What will you do with your child as a result of this session?
- What did you find interesting tonight?

