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(1)

## Addition with Base Ten Blocks

Do these activities with a partner and record your answers on your own paper.


1. Using your base ten blocks form three thousand seven hundred twenty four.
2. Using only the base ten blocks, add: seventy-eight and sixty-five.
3. Using only the blocks, add: nine, seventy-three, three hundred sixteen, and one thousand six hundred ninety seven.
4. Using only the blocks, add: two hundred thirty six and five hundred eighty nine.
5. Do activity 4 again but use only pencil and paper.
6. Discuss with your partner how the thinking in activity 5 relates to the process in activity 4? Record your thoughts.

## Challenge Activity:

I bought a bag with five hundred blocks in it. I dropped the bag and the blocks scattered over the place. My friends Bill, Rachel, and Josh started gathering them up. Bill found one hundred thirty nine blocks, Rachel found fifty-seven, and Josh found two hundred seventy. Did they find them all? If not, how many are still lost?

## Addition with a Counting Board

Do these activities with a partner and record your answers on your own paper.


1. Represent, on your counting board, the number thirteen thousand two hundred sixty five.
2. Using only your counting board and beans add the numbers, one thousand eight hundred twenty nine and four hundred eighty three. Keep track of the steps you made in doing the addition. Show any exchanges that you make with your beans.
3. Do Activity 2 using pencil, paper and numbers.
4. Discuss with your partner how the thinking in activity 2 relates to the process in activity 1? Record your thoughts.
5. Using the counting board, find out how many hours there are in a week.

## Challenge Activity

Using the counting board only, find out how many eggs there are in thirteen cartons. Each carton contains a dozen eggs.

## Addition Principles

Do these activities with a partner and record your answers on your own paper.

## Activity 1

True or False: $A+B=B+A$.
a. What does this mean?
b. How can you test it?
c. What is the most basic way you can think to explain it to a child?

## Activity 2

The symbol "<" means "less than".
True or False: Suppose that $A<B$.
Is $A+C<B$ ?
a. What does this mean?
b. How can you test it?
c. What is the most basic way you can think to explain it to a child?

Cut out around edges and cut on dotted lines to make 40 units:


Cut out around edges to make 20 rods:



## Base Ten Template - Flats

Cut out around edges to make 6 flats:


## Base Ten Template - Cube

Cut out around edges, fold on dotted lines, and tape edges together:



## Number Activities with Base Ten Blocks

In your day to day experience:
Notice exchanges that we do in everyday life. Jot down ideas that come to you. For instance, when talking about measurement, I might exchange 2 pints for a quart, or 4 quarts for a gallon.

With younger children:
A) Do the session's activities using blocks and counting boards with your children. Ask them to explain their thinking.
B) Do addition with Whole Numbers.

With older children:
A) Do the session's activities using blocks and counting boards with your children. Ask them to explain their thinking. Have them do the activity with paper and pencil and ask them to describe how paper and pencil relates to the other two methods.
B) Do addition with Whole Numbers.

## Addition with Whole Numbers

1. Complete any activities (including the challenge activities) on the handouts that you did not complete during class.
2. Use your Counting Board

I have five hundred dollars to spend from a gift. I buy an entertainment system for two hundred ninety seven dollars and a warm parka for eighty-nine dollars. I want to buy warm up suits for thirty-six dollars each.
a. How many can I buy?
b. How much money will I have left over?
3. True or False: If $A<B$, then $A+A<B+B$.
a. What does this mean?
b. How can you test it?
c. What is the most basic way you can think to explain it to a child?

## Connecting Multiplication Models

Do these activities with a partner.

1. Multiply on your paper:
2. Using only the base ten blocks, model $14 \times 6$. Draw a picture of that model. Record your process with numbers. Drawing:
3. Now imagine that you have a room in your house that is $14^{\prime} \times 6^{\prime}$. Draw this room on the grid paper. Label the sides of the room. How many tiles (represented by the small squares) will fit on the floor of this room.

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4. Discuss with your partner how the process in activity 1 relates to the models in activities 2 and 3. Record your thoughts.

## Multiplication Principles

Do these activities with a partner and record your answers on your own paper.

## Activity 1

True or False: $A \times B=B \times A$.
a. What does this mean?
b. How can you test it?
c. What is the most basic way you can think to explain it to a child?

## Activity 2

Exponential notation $A \times A$ is often written as $A^{2}$. The two in the upper right is called an exponent. We say, " $A$ squared". Similarly $A \times A \times A$ is written as $A^{3}$ and $A \times A \times A \times A$ is written as $A^{4}$.

True or False: If $A>4$ then $A^{2}>20$. (Remember that ">" means "bigger than".)
a. What does this mean?
b. How can you test it?
c. What is the most basic way you can think to explain it to a child?

## Bringing Mathematics Home - 2

## $4 \times 5$

Multiplication Activities

1. Try some of the activities from Session 2 with your children.

With older children:
Have your child multiply $14 \times 6$.
With younger children:
Have your child multiply $8 \times 7$.
a. What was easy for your child?
b. Was your child able to explain his/her method of multiplying?
2. Have fun with your children this week making up imaginative stories that involve multiplication situations. Be ready to share some of your favorites.

## Multiplication with Whole Numbers

1. For the problem: $35 \times 13$ :
a. Multiply $35 \times 13$.
b. Show the multiplication through a model.
c. Make up a word problem for $35 \times 13$.
d. Show the connections between the model and the traditional way of multiplying.
2. Which deal would you accept and why?
a. You are given a thousand ten dollar bills every day for a month.
b. You are given a penny on the first day of the month and the value of that penny doubles every day for a month.
3. True or False: Mathematicians use parenthesis to indicate which portion of the problem should be done first. Keeping that in mind, is

$$
A \times(B+C)=(A \times B)+(A \times C) ?
$$

## Do this problem:

$$
4 \longdiv { 9 7 5 }
$$

Share the money and record your steps.

| 1st <br> person | 2nd <br> person | 3rd <br> person | 4th <br> person | Money <br> Used | Exchanges |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Notes:

## Applications of Division

There are three ways to represent the answer to these division problems: 243 R3, 243.75 , or $2433 / 4$ Sometimes one way is more helpful than another. In the following situations, which is most helpful? Do this activity with a partner and choose the form of the answer that would fit each situation. Explain your reasoning.

1. The T-ball league received 975 pieces of candy on our Halloween parties. Each of the 4 teams is planning their own party. How many pieces should each team receive if the candy is shared equally?
2. The $T$-ball league raised $\$ 975$ selling candy. If there are 4 teams in the league, how much does each team get to spend on uniforms?
3. Dominoes donated 975 pizzas to our T-ball league. If there are 4 teams, how many pizzas does each team get?

## Challenge Activity

Make up problems of your own.
Make up one problem that could use a whole number remainder, one that could use a decimal and a third that could use a fraction.

## Division Principles

1. Explore the following idea by looking for examples and counter-examples of the statement.

True or false: If $A>B$, and $B>C$, then $A \div B>A \quad C$.
2. Explore the following situation.

Find as many numbers as you can that:
When the number is divided by 2 , it leaves a remainder of 1 .
And also, when the same number is divided by 3 , it leaves a remainder of 2 .

## Money Template for Bills

Copy the following to make $20 \$ 1$ bills, $20 \$ 10$ bills and $10 \$ 100$ bills for each group of 4 .


## Money Template for Coins

Copy the following to make 50 dimes and 30 pennies for each group of 4 .


## Bringing Mathematics Home - 3



## Money Activities

Try some of the activities from Session 3 with your children.

## With younger children:

Do some equal sharing with money. For instance, take out $\$ 20$ dollars and have them share it 4 ways. Ask them how they might record what they have done and observe their thinking process about the problem. They might draw a picture to record the problem, or use numbers or tally marks. Bring in some of their work to share.

## With older children:

Do the same problem that we did in Session 3. Be ready to share their thoughts about it with the class.

## Challenge:

For children that are ready for a challenge:
Do the Division Principles handout with your children and bring in their work.

## Division with Whole Numbers

1. Divide: $5 \longdiv { 6 5 8 }$
a. Do the problem with long division.
b. Show it as repeated subtraction.
c. Express the quotient as a whole number with a remainder, a fraction and a decimal.
d. Write a word problem that would represent this situation.
2. True or false: If $A>B$, and $B<C$, then $A \div B>A \div C$.


## Concepts of Multiples

1. Demonstrate how to find the answer to the hot dog question using Cuisenaire ${ }^{\circledR}$ rods.
A. Lay the different colored rods out according to length. Using the smallest rod, figure and record the length of each color.

- White
- Red
- Green
- Yellow
- Purple

B Form a train using the 8 unit rods and another train using the 10 unit rods. Build the two trains until their lengths coincide. Record it in a drawing.

- How many rods did it take?
- How does this relate to our hot dog question?
- Predict when they will next coincide. Check your prediction by building it.
- Name some other amounts where the hot dogs and buns would match.
- Out of the numbers that we have been using, name as many multiples and divisors as you can. Record each, stating the two numbers that form the relationship. Example: 5 is a divisor of 10.10 is a multiple of 5 .

2. Margie and Bernadette want to buy a stereo for their room. They have decided that they will work after school. Margie earns $\$ 7$ an hour and Bernadette earns $\$ 4$ an hour. Margie wants to work only long enough to match Bernadette.
A. How long will each girl need to work so that they each contribute the same amount?
B. What is that amount?

Work with a partner to solve this problem. Be ready to defend your answer.
3. There are some pennies on my table. If I share them equally among four people there are none left over. If I share them equally among six people there are none left over.
A. How many pennies are on the table?
B. Is there only one answer to this question? Explain.
4. Four students are asked to count how many chairs there are in the room. They are told there are more than one hundred but fewer than two hundred chairs in all but they are not sure how many. One student counts them by twos; that is $2,4,6,8$, etc. Another counts by threes; that is $3,6,9$, etc. Another counts by fours; that is $4,8,12$, etc. Another counts by fives; that is $5,10,15$, etc. In each case all the chairs are counted with no remainders; that is no chair is left over.
A. How many chairs are there?
B. Is there only one answer to this question?

## Concepts of Divisors

1. Build as many rectangles as possible with 24 pennies. You must use all of the pennies and they must make a rectangle. Record the dimensions of each rectangle.

24: $\quad$| Length |  |
| :--- | :--- |
| Width |  |

2. Build as many rectangles as possible with 36 pennies. You must use all of the pennies and they must make a rectangle. Record the dimensions of each rectangle.

36: $\quad$| Length |  |
| :--- | :--- |
| Width |  |

3. Jocelyn is a quilter. A Museum has asked Jocelyn to make a very large quilt for the entry way. She has a 24 ' by 36 " piece of cloth. She wants to cut out square of equal size and use all the material. Use the samples of cloth on the next page to find different sized squares that he can use. See the example of using a $2^{\prime} \times 2^{\prime}$ square. Notice that the squares are all equal in size and take up the entire cloth with nothing left over. List all of the possibilities.
4. Go back to question \#1 and \#2. Circle the lengths and widths that 36 and 24 have in common. Describe the relationship between these numbers and the size of the squares that you found in question \#3.
5. Greatest Common Divisor
A. Identify the Greatest Common Divisor of 24 and 36.
B. In your own words, define Greatest Common Divisor.
(You might want to include an example)
6. Find the greatest common divisor of the following pairs of numbers. After you have found it one way, check it with another way.
A. 12 and 18
B. 14 and 35
C. 8 and 15

Pick 2 numbers of your own to explore.

## Principles of Divisors and Multiples

1. True or False:

If you know one divisor of a number, you can immediately find another. Explain your reasoning.
2. True or False:

Multiples of 6 are always even numbers.
Explain your reasoning.
3. You will remember from session 2 that the product is the answer to multiplication.

True or False:
A. The product of two numbers is a common multiple of both numbers.

Explain your answer.
B. The product of two numbers is a smallest common multiple of both numbers.

## Bringing Mathematics Home - 4

## Clapping on Multiples Game



Try some of the activities from Session 3 with your children.
Play the following game with your children:
Clapping on multiples:
In this game, people sit in a circle. A single digit number is picked (for example three). The first person says one, the second says two, but the third person must clap instead of saying three, because three is a multiple of three. The fourth person says four, the fifth says five and the sixth has to remember not to say six, but to clap. This continues until someone makes a mistake and calls out a number that is a multiple of three, or the groups reaches a decided upon goal (for example, 30).
*Note: The number that you choose should not be the same number as people in your circle.

Option 1; Add the rule that if a number contains your chosen digit, you must clap on that number as well. So if you have chosen three, you would clap on 13 because it contains a three.
Option 2: Pick a number like 24 and clap on all of the divisors of that number.

## For older children:

Play the factor game several times with your children. Children develop strategies for game only after they have played several times. Ask your children what they think is the best first move for the game and why they think it. This is not a time to teach them the best first move, just record and notice their thinking.

## The Factor Game

Players: 2 or 2 teams

## Directions:

- Player 1 chooses a number (25). Player one receives 25 pts for their team.
- Player two now identifies all of the factors for 24 that have not been used in a prior turn. So player two gets $1+5$, or 6 pts. 1, 5, and 25 are crossed out.
- Player two now chooses a number that has at least on factor that is not crossed out (30).
- Player two get 30 pts.
- Player one identifies $2,6,10$ and 15 as factors that have not been crossed out. Player one gets 33 pts.
- Play continues until there are not numbers remaining that has factors that have not been used.

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 |
| 26 | 27 | 28 | 29 | 30 |

Team 1

Team 2

## Divisors (Factors) and Multiples of Whole Numbers

1. Find a number that has more than nine divisors.
2. I have some pennies on the table.

- When I split them up in two piles, 1 penny is left over.
- When I split them up into three piles, 2 pennies are left over.
- When I split them up into five piles, 3 pennies are left over.

How many pennies are there?
3. True or False:

The factors of even numbers are always even.
1.

2.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

3. 

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

4. Why are no more numbers sieved out of the hundreds chart after the multiples of 7 ?

Challenge: If we extended the table to 150 numbers, which other prime multiples would we need to cross out? How about to 200? Explain your answer.

## Prime Factorization

In order to find the prime makeup of numbers, it is handy to use a factor tree.

1. Use a factor tree to determine the prime factorization of the following numbers:
36
57

91
100

99
288
2. Use a factor tree to determine the prime factorization of the following numbers: 582 2860

966
25575
3. Pick 2 numbers of your own to explore.

## Principles of Prime and Composite Numbers

1. True or False: In Eratosthenes' sieve there is no need to cross out multiples of composite numbers. Explain your reasoning.
2. The numbers from 2-1000:
A. In a listing of the numbers from 2-1000, how many numbers are crossed out in step 1 (crossing out all of the multiples of 2 that are greater than 2)?
B. How many new numbers are crossed in step 2 (crossing out all of the multiples of 3 that are greater than 3)? Explain your reasoning.
3. Finding greatest number of factors:
A. For numbers less than 50, which number has the most factors in its prime factorization?
B. Is there more than one answer?
C. What would be the next number with the same amount of factors in its prime factorization?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

## Prime Number Activities

## With younger children:

The Mystery of Numbers:
The purpose of this activity is to introduce young children to the idea of prime numbers without introducing the definition or vocabulary.

- Use the 1-50 chart and do an investigation with your child.
- Tell them that there are some special numbers that feel left out because people do not use them when they are counting by $2 s, 3 s$, etc.
- Tell them that you are going to find these mystery numbers together.
- Ask your child to circle the 2, and then count by 2s. As your child counts by 2s, cross out those numbers.
- Ask your child to circle the 3 . When counting wheels on a tricycle, people count by 3 because there are 3 on each trike. Have your child count by 3s. As your child counts by 3 s , cross out those numbers.
- Ask your child to circle the 5 . When counting fingers, people count by 5 because there are 5 on each hand. Have your child count by 5s. As your child counts by 5s, cross out those numbers.
- Tell them that when people watch football, they count the touchdowns, which are worth 7 points each. Circle the 7 and then count touchdowns together, crossing off those numbers. Notice the numbers that are left. They are the mystery numbers.


## With older children:

Do the Sieve of Eratosthenes with your child.
Try some of the factor trees.

## Calculator activity:

Pick a three-digit number; multiply it by 7 then by 11 then by 13. Do this several times. What is the pattern in your answer? I wonder why this happens.

## Prime and Composite Numbers

1. Extend your research for prime numbers to the numbers from 1 to 300 .
A. Twin primes are prime numbers that are only 2 apart, like 5 and 7 or 11 and 13 .

Find as many twin primes as you can.
B. List any other patterns that you see.
2. Find the largest prime number that you can and be prepared to share it with the class.
3. True or False: The factors of even numbers are always even. Use examples and counter examples to explain your reasoning.
4. Let the letter $n$ represent any whole number. Substitute different numbers for $n$ to see what kind of patterns you find.

What kinds of numbers are produced by $6 n-1$ ? ( 6 times $n-1$ )

## Prime Factorization Using Exponential Notation

1. Use a factor tree to determine the prime factorization of the following numbers and then write each prime factorization in exponential form.

4
8

125
243
2. Use a factor tree to determine the prime factorization of the following numbers and then write each prime factorization in exponential form.
100
54

175
196

## Applications of Growth

Aunt Bessie ( 75 years old) is drawing up her will and wants to put you in it. She has said.
"My dear, you must not tell anybody but you are my favorite niece and so I am offering you a special thing in my will. You may choose which you would rather have:

- one million dollars today,
- five thousand dollars every month for the rest of my (Aunt Bessie's) life, or
- I will give you 2 cents today, 4 cents tomorrow, 8 cents the next day and keep doubling the new amount for a total of 28 days.

It is so important that you keep this secret that I will have to withdraw the offer if you tell anyone."

1. Without doing any mathematics which would you choose?
2. Do the math. Figure out all three options including any assumptions that you make. Be prepared to share your reasoning.

## Principles of Exponential Notation

1. Is this statement always true, sometimes true, or never true?

$$
3^{n}>3 \times n
$$

A. What is your discovery? Show at least three examples and/or three counter examples to prove your reasoning.
B. What if the 3 were a different number? Is your answer still the same? Explain your reasoning.
2. Is this statement always true, sometimes true, or never true?

$$
\frac{a^{5}}{a^{3}}=a^{2}
$$

What is your discovery? Show at least three examples and/or three counter examples to prove your reasoning.
3. Is this statement always true, sometimes true, or never true?

$$
a^{2} \times a^{3}=a^{5}
$$

What is your discovery? Show at least three examples and/or three counter examples to prove your reasoning.

## Bringing Mathematics Home 6



## The Sky is the Limit Game

1. Try out the activities of this session with your child.
2. What is different about the growth of a number when adding, multiplying, and using exponents? The following game is designed to help children develop an intuitive sense about growth operations.

Game: The Sky is the Limit

## Materials

- Set of 9 index cards for making the playing cards (See page BLM 36.3 for instructions)
- Number cubes or cards numbered 1 to 6


## Preparation

- Cut the index cards in half so that you have 18 cards
- Place the following number sentences on each card

Number of Players
Two or three teams

See game directions on BLM 36.2

## The Sky is the Limit Directions



## Directions (Cooperative)

- Shuffle the cards and place all of them face down in a stack.
- Draw the first 5 cards off the stack and place them face up.
- One player rolls the numbered cube.
- Use this number as the number for the first round.
- Each player guesses which card will produce the largest result.

If you are playing with a child who is 8 years old or younger, research shows it is best to let them "win" often. Let your child choose first, if they choose incorrectly, then you might choose incorrectly as well. Act surprised with the correct answer.

- Afterwards compute to see if you were right.
- If either of the players guess it right, that card is put in the discard pile.
- A new card is drawn to replace the one that is removed.
- The second player rolls the numbered cube and use this number as the number for the second round.
- This process is repeated.


## Directions (Competitive)

- Shuffle the cards and deal 5 to each person. Place the remaining cards face down in a draw pile.
- The dealer rolls the numbered cube.
- Use this number as the number for the first round.
- The dealer looks at his cards and chooses what he feels is the largest value. He places this card face up on the table first and announces its value.
- Then the other player looks at his cards and chooses what he feels is the largest value. He places this card face up on the table and announces its value.
- The player with the largest value wins both cards.
- Both players draw a new card from the draw pile.
- The second player rolls the numbered cube and uses this number as the number for the second round.
- In this round the second player plays first.
- This process is repeated until all the cards are played.
- The winner is the player who has the most cards in their winning pile.


## The Sky is the Limit Cards

## Preparation of Cards

Cut the index cards in half to create 18 cards. Write one sentence on each card. For older children you may use the expression while for younger children you will want to use the words.

| number $\times$ number (number) ${ }^{2}$ $n^{2}$ | number $x$ number $x$ number (number) ${ }^{3}$ $n^{3}$ | $\begin{gathered} \text { number } \times \text { number }-5 \\ (\text { number) } \\ n^{2}-5 \end{gathered}$ |
| :---: | :---: | :---: |
| $\begin{gathered} 2 \times \begin{array}{c} \text { number } \\ 2 n+1 \end{array}+1 \end{gathered}$ | $\begin{gathered} \text { number }+5 \\ n+5 \end{gathered}$ | $\begin{gathered} 5 \times \text { number }+2 \\ 5 n+2 \end{gathered}$ |
| $\begin{gathered} \text { number }+10 \\ n+10 \end{gathered}$ | $\begin{gathered} \text { number } \times 10 \\ 10 n \end{gathered}$ | $\begin{gathered} 2 \times \text { number }+10 \\ 2 n+10 \end{gathered}$ |
| $\begin{gathered} 2 \times \text { number }-1 \\ 2 n-1 \end{gathered}$ | $\begin{gathered} 5 \times \text { number }-10 \\ 5 n-10 \end{gathered}$ | $\begin{gathered} 2 \times \text { number }+2 \times \text { number } \\ 2 n+2 n \end{gathered}$ |
| $\begin{gathered} \text { number }+20 \\ n+20 \end{gathered}$ | $\begin{gathered} \text { number }+8 \\ n+8 \end{gathered}$ | $\begin{gathered} \text { number }+3 \\ n+3 \end{gathered}$ |
| $\begin{gathered} 5 \times \text { number }-2 \\ 5 n-2 \end{gathered}$ | $\begin{gathered} 10 \times \text { number }-10 \\ 10 n-10 \end{gathered}$ | $\begin{gathered} 5 \times \text { number }-5 \\ 5 n-5 \end{gathered}$ |

Note: Variations to the game can be achieved by creating new expressions with your child that are appropriate to what they are learning in school.

## Exponential Notation

1. I was so excited with what Aunt Bessie told me that I couldn't resist telling my sister and my best friend the next day. But I told them never to tell anyone and I never told anyone else so I was certain my secret was safe. What I didn't know was that on the second day my sister and my best friend each told two people. Each told two different people and never told anyone else. And so it went that as each person learned of the secret they told two other people.
A. How many days will it take before 100 people know the secret?
B. How long do you think it took before Aunt Bessie found out?
C. How many days will it take before 1000 people know the secret?

Bonus: How long will it take until every person in the world knows the secret? Assume that no one ever hears the secret twice.
2. Is this statement always true, sometimes true, or never true?

$$
2^{4}>4^{2}
$$

A. What is your discovery? Show at least three examples and/or three counter examples to prove your reasoning.
B. What if the 2 and 4 were different numbers. Is your answer still the same? Explain your reasoning.

## Powers of Ten

| Multiply | Power <br> of 10 | Answer | Number <br> of Zeros |
| :--- | :---: | :---: | :---: |
| $10 \times 10$ | $10^{2}$ | 100 | 2 |
| $10 \times 10 \times 10$ |  |  |  |
| $10 \times 10 \times 10 \times 10$ |  |  |  |
| $10 \times 10 \times 10 \times 10 \times 10$ |  |  |  |
| $10 \times 10 \times 10 \times 10 \times 10 \times 10$ |  |  |  |
| $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ |  |  |  |
| $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ |  |  |  |

## Powers of Ten

## TI

1. What patterns do you notice?
2. Write $1,000,000$ as a power of ten.
3. Multiply $1,000,000$ times $1,000,000$.
a. Express the answer as a power of ten.
b. Write $1,000,000$ times $1,000,000$ as the multiplication of powers of 10 .
c. What do you notice? What is the shortcut method for multiplying powers of 10?
d. Test your suspicion with the following situations:

- $10,000,000 \times 10,000$
- $1,000 \times 100,000,000$
- 100,000,000,000 $\times$ 10,000,000,000.


## Scientific Notation <br> (2) e)

1. Convert the following to scientific notation:
2,000,000
400, 000
60
90,000,000
2. Light travels at a speed of approximately 300,000,000 meters per second. There are a little less than 90,000 seconds in a day. Using these estimations, approximately how far has light traveled in a day?
A. What was different about this problem?
B. How does your calculator deal with this new challenge?

3 Practice using scientific notation with the following numbers:
230,000
6.800,000
$3.5 \times 10^{6}$
$7.432 \times 10^{2}$
4. Is this statement true or false?
A. How do you multiply in scientific notation?
B. Multiply the following numbers (the star indicates multiplication):
$4 \times 10^{6} * 2 \times 10^{3}$
$2 \times 10^{3} * 2 \times 10^{3}$
$3 \times 10^{4}$ * $3 \times 10^{4}$

## Estimating with Scientific Notation



1. Light travels 186 thousand miles per second. A year contains $31,536,000$ seconds. A light year is the distance light travels in one year. How many miles does light travel in a year? To estimate this, round each of the numbers off and use your knowledge of scientific notation to estimate how many miles it travels.
A. Use a calculator to find the answer. How close were you?
B. The closest star to our sun is Alpha Centauri. It is 4.3 light years away. Approximately how far away is this?
2. Light travels at 186,000 miles per second. It is known that it takes light 500 seconds to reach Earth from the Sun. How far away is the sun from Earth? Again, use scientific notation to estimate the distance.

Use a calculator to find the answer. How close were you?
3. If the national debt was represented in pennies, and those pennies were stacked on top of each other, the stack of pennies would be $394,570,707$ miles high. Wow! That is a lot of pennies! There are 5280 feet in one mile. Use scientific notation to estimate the amount of pennies in this stack.

Use a calculator to find the answer. How close were you?

1. Explain why it is correct to add the exponents of 10 's when you multiply numbers in scientific notation.
2. Is it equally correct to subtract exponents when you divide numbers that are written in scientific notation?
3. True or False: Even though we may not know what it is, there is a largest number. Explain your reasoning. <br> \title{
Large <br> \title{
Large Number Number Activity
} Activity
}

## Younger children

Exploration: This week your child will do an exploration. They will think of the largest number they can write. This will be posted on the refrigerator. They will ask their friends (not teachers) if there is a larger number. Each time they find a larger number it will also be posted on the refrigerator. Your job in this exploration is to watch the process that your child goes through. You need to make sure that older brothers and sisters do not interfere with this process.

- Tomorrow ask your child to write the largest number that he/she can think of.
- Admire how large that number is and post it on your refrigerator. Tell your child that this week we will explore to see if this is the largest number.
- Have your child ask their friends if there is a larger number.
- If they find a larger number post it.

Be ready to share this process with other people in your group.

## My largest number is:

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## Scientific Notation Activity

## Older children

Listed below is the distance in km . of each planet from the Sun. In the table provided, order the planets from closest to farthest from the sun. Then write each distance in scientific notation.

| Jupiter | $778,300,000$ | Saturn | $1,427,000,000$ |
| :--- | :--- | :--- | :--- |
| Uranus | $2,870,000,000$ | Neptune | $4,497,000,000$ |
| Mercury | $57,900,000$ | Venus | $108,200,000$ |
| Earth | $149,600,000$ | Mars | $227,900,000$ |
| Pluto | $5,900,000,000$ |  |  |


| Planets distance from the sun | Distance in scientific notation |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Challenge:

Make a model or picture that shows each planet. Estimate how far apart they are from each other. Write an explanation about how you decided where to put them.

## Estimating and Writing Large Numbers

## 1,000,000,000,000

1. Practice in estimating:
A. Which is closest to being true? One trillion hours ago ...

- The American revolution was being fought.
- Christ was living.
- A meteor struck Arizona (about fifty thousand years ago)
- Dinosaurs ruled the earth (about one hundred million years ago).
B. Make a guess.
C. Now estimate it using scientific notation.
D. What did you find?

2. In the movie "Rush Hour", one million dollars were stolen. The money was all in $\$ 50$ bills and it was packed into two suitcases. Is this possible? Explain why or why not.
3. Bill Gates is supposed to be worth $\$ 80$ billion dollars. If he gave each family in your city an equal share of his money, what would you be able to do?
A. Buy a nice outfit.
B. Buy the car of your dreams.
C. Buy the house of your dreams.
D. Retire and never work again.

## Square Numbers

1. Write the first ten square numbers.
2. Some of you may recall the following nursery rhyme.

As I was going to St. Ives, I met a man with seven wives, Each wife had seven sacks, Each sack had seven cats,
Each cat had seven kits.
Kits, cats, sacks, and wives, How many were going to St. Ives?

How are perfect squares used to compute the number going to St. Ives?
3. Add consecutive odd numbers together and what do you find?
A. Complete the following table:

| Consecutive odd numbers | Sum |
| :---: | :---: |
| 1 |  |
| $1+3$ |  |
| $1+3+5$ |  |
| $1+3+5+7$ |  |
| $1+3+5+7+9$ |  |
| $1+3+5+7+9+11$ |  |
|  |  |

B. What did you discover about the sum?
C. Write in words how you would find the sum of the first 4 consecutive odd numbers?
D. Is there a short cut for finding these sums?

## Square Numbers

4. Multiply any two consecutive odd numbers together and what do you find?
A. Begin by completing the table.

| Consecutive odd numbers | Product <br> (Result by multiplying) |
| :---: | :---: |
| $3 \times 5$ |  |
| $5 \times 7$ |  |
| $7 \times 9$ |  |
|  |  |
|  |  |
|  |  |
|  |  |

B. Describe the pattern.
5. Multiply any two consecutive even numbers together and what do you find?
A. Begin by completing the table.

| Consecutive odd numbers | Product <br> (Result by multiplying) |
| :---: | :---: |
| $4 \times 6$ |  |
| $6 \times 8$ |  |
| $8 \times 10$ |  |
|  |  |
|  |  |
|  |  |
|  |  |

B. Describe the pattern.

## Number Tricks

1. Magic Numbers
A. Pick a number between 1 and 9
B. Double the number
C. Add 14
D. Divide the result by 2
E. Subtract your original number
$\qquad$

F. What do you have left?

- Check with your group to see what number they started with and ended with.
- What do you all have in common?
- Will this always be true? Try some other numbers larger than 9.
- Examine the instructions, use tiles, or draw pictures in order to figure out why this happens.


## G. Challenge:

- Rewrite the instructions so that the resulting number is 11.
- Write your own magic number puzzle.

2. How old are you?
A. Write the number of the month you were born. (Since I was born in March, I would use a 3)
B. Double the number (take it times 2)
C. Add 5 to the result $\dagger$
D. Multiply by 50
E. Subtract 250 $\qquad$
F. Add the present year (Example: 2003) $\qquad$
G. Subtract the year of your birth (Example 1981) $\qquad$
H. Circle the last two digits.
I. The circled number is your age on your birthday this year!
$J$. The uncircled number should be your birth month!

## Number Tricks

3. Follow the series of steps and what do you find?

|  | Series of Steps | Trial 1 | Trial 2 | Trial 3 | Trial 4 |
| :---: | :--- | :---: | :---: | :---: | :---: |
| 1 | Write down any four digit number | 4567 |  |  |  |
| 2 | Switch the first and last digit only <br> and write down the new number | 7564 |  |  |  |
| 3 | Subtract the small number from <br> the larger | 7564 <br> -4567 |  |  |  |
| 4 | Divide your answer by the <br> difference between the first and <br> last digit | $7-4=3$ <br> Divide <br> by 3 |  |  |  |
| 5 | Final result |  |  |  |  |

A. What is the pattern?
B. Challenge: Does it work for a 2, 3 or 5 digit numbers?
C. Super Challenge: How can you explain this?

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## Younger children

Do the following activity with manipulatives. Beans work well.

|  | Series of Steps | Trial 1 | Trial 2 | Trial 3 |
| :---: | :--- | :--- | :--- | :--- |
| 1 | Pick a number |  |  |  |
| 2 | Add 5 |  |  |  |
| 3 | Subtract 2 |  |  |  |
| 4 | Subtract your original number |  |  |  |

## Older children

1. Do any of the number tricks you did in class with your children.
2. Here is another number trick to try with your children.
a. Multiply $12345679 \times 18$
b. Multiply $12345679 \times 81$
c. Find: $12345679 \times 36$
d. Find: $12345679 \times 63$
e. Can you guess what $12345679 \times 45$ is?
f. How about $12345679 \times 72$ ?
g. What is going on?
